■ Lemma 4.3: Computations for Claim 1, Case B

In[1]:=
```
SetDirectory["~/writing/WIP/KappaLib/"];
<< KappaLib.m
```

```
KappaLib v1.1
```

In[3]:=
```
Ax = DiagonalMatrix[{a1, a2, 0}];
Bx = emGeneralSymmetric3x3["B"];
Cx = emGeneral3x3["C"];
Dx = Transpose[Cx];
```

In[7]:=
```
eq1 = Flatten[Cx.Cx + Ax.Bx + IdentityMatrix[3]];
eq2 = Flatten[Bx.Cx + Transpose[Cx].Bx];
eq3 = Flatten[Cx.Ax + Ax.Transpose[Cx]];
eqs = Join[Join[eq1, eq2], eq3];
```

In[11]:=
```
Simplify[eqs] // MatrixForm
```

Out[11]//MatrixForm=

$$\begin{pmatrix}
1 + a1\,B11 + C11^2 + C12\,C21 + C13\,C31 \\
a1\,B12 + C11\,C12 + C12\,C22 + C13\,C32 \\
a1\,B13 + C11\,C13 + C12\,C23 + C13\,C33 \\
a2\,B12 + C11\,C21 + C21\,C22 + C23\,C31 \\
1 + a2\,B22 + C12\,C21 + C22^2 + C23\,C32 \\
a2\,B23 + C13\,C21 + C23\,(C22 + C33) \\
C11\,C31 + C21\,C32 + C31\,C33 \\
C12\,C31 + C32\,(C22 + C33) \\
1 + C13\,C31 + C23\,C32 + C33^2 \\
2\,(B11\,C11 + B12\,C21 + B13\,C31) \\
B11\,C12 + B22\,C21 + B12\,(C11 + C22) + B23\,C31 + B13\,C32 \\
B11\,C13 + B23\,C21 + B12\,C23 + B33\,C31 + B13\,(C11 + C33) \\
B11\,C12 + B22\,C21 + B12\,(C11 + C22) + B23\,C31 + B13\,C32 \\
2\,(B12\,C12 + B22\,C22 + B23\,C32) \\
B13\,C12 + B12\,C13 + B23\,C22 + B22\,C23 + B33\,C32 + B23\,C33 \\
B11\,C13 + B23\,C21 + B12\,C23 + B33\,C31 + B13\,(C11 + C33) \\
B13\,C12 + B12\,C13 + B23\,C22 + B22\,C23 + B33\,C32 + B23\,C33 \\
2\,(B13\,C13 + B23\,C23 + B33\,C33) \\
2\,a1\,C11 \\
a2\,C12 + a1\,C21 \\
a1\,C31 \\
a2\,C12 + a1\,C21 \\
2\,a2\,C22 \\
a2\,C32 \\
a1\,C31 \\
a2\,C32 \\
0
\end{pmatrix}$$

- **Since a1,a2 != 0, it follows that C31,C32,C11,C22 = 0.**

In[12]:=
```
subs = {};
subs = Append[subs, C31 → 0];
subs = Append[subs, C32 → 0];
subs = Append[subs, C11 → 0];
subs = Append[subs, C22 → 0];
Simplify[Union[eqs //. subs]] // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 \\ a1\ B12 \\ a2\ B12 \\ 2\ B12\ C12 \\ 2\ B12\ C21 \\ a2\ C12 + a1\ C21 \\ B11\ C12 + B22\ C21 \\ 1 + a1\ B11 + C12\ C21 \\ 1 + a2\ B22 + C12\ C21 \\ B11\ C13 + B23\ C21 + B12\ C23 + B13\ C33 \\ B13\ C12 + B12\ C13 + B22\ C23 + B23\ C33 \\ 2\ (B13\ C13 + B23\ C23 + B33\ C33) \\ a1\ B13 + C12\ C23 + C13\ C33 \\ a2\ B23 + C13\ C21 + C23\ C33 \\ 1 + C33^2 \end{pmatrix}$$

- **Equation 1+C33^2 = 0 has no real solution for C33. Thus rank A can not be 2.**