In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
```

```
Loading KappaLib v1.2
```

- **Metaclass IV:**

In[3]:=
```
kappa = emMatrixToKappa[
```

$$\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & a4 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & a4 & 0 & 0 & a3 \end{pmatrix}$$

```
];
```

- **By Theorem 3.5 we may assume that a2 = a1 and b2 = b1**

In[4]:=
```
sub = {a2 → a1, b2 → b1};
kappa = kappa //. sub;
```

- **Let us also assume that a1 = a3**

In[6]:=
```
sub = Append[sub, a3 → a1];
kappa = kappa //. sub;
```

- **Medium characteristics**

In[8]:=
```
emKappaToMatrix[kappa] // MatrixForm
FullSimplify[emDet[kappa]]
Simplify[emTrace[kappa]]
```

Out[8]//MatrixForm=

$$\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a1 & 0 & 0 & -b1 & 0 \\ 0 & 0 & a1 & 0 & 0 & a4 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b1 & 0 & 0 & a1 & 0 \\ 0 & 0 & a4 & 0 & 0 & a1 \end{pmatrix}$$

Out[9]= $(a1 - a4)\,(a1 + a4)\,\left(a1^2 + b1^2\right)^2$

Out[10]= 6 a1

- **Define coefficients in Theorem 5.1:**

In[11]:=
```
rho = (a4^2 + b1^2) / 2;
mu = -a1;
lambda = b1^2;
```

```
Abivector =
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

```
Bbivector =
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix};$$

```
Simplify[Abivector + Transpose[Abivector]]
Simplify[Bbivector + Transpose[Bbivector]]
```

Out[16]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

Out[17]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

- **Verify claim:**

In[18]:=
```
eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
RHS = -lambda emIdentityKappa[] +
   emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
Union[Flatten[Simplify[LHS - RHS]]]
```

Out[21]= {0}

- **Note:**
  *** kappa + mu Id is trace-free in this case**
  *** lambda > 0**
  *** rho, A, B are all non-zero**

In[22]:=
```
emTrace[eta]
```

Out[22]= 0

Solvability of equation for D

- **Define constants alpha, beta, gamma that appear in definition of algebraically decomposable medium**

In[23]:=
```
alpha = lambda + mu^2;
beta = mu;
gamma = 1;
```

- **Explicitly verify that kappa is algebraically decomposable:**

In[26]:=
```
LHS = alpha emIdentityKappa[] +
   beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
RHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
eqs = Union[Simplify[Flatten[(LHS - RHS)]]]
```

Out[28]= {0}

- **Existence of D:**

  Note: Theorem 3.5 ensures that a4 != 0.

In[29]:=
```
Dbivector = 1 / a4 Abivector;
(* contract kappa with a bivector from the left *)
contract[biv_, kappa_] := Table[
   1 / 2 Sum[
     biv[[i]][[j]] emReadNormal[kappa, a, b, i, j]
     ,
     {i, 1, 4}, {j, 1, 4}
    ]
   ,
   {a, 1, 4}, {b, 1, 4}
  ]
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[Simplify[dLHS - dRHS]]]
```

Out[33]= {0}

- **Note: Theorem 3.5 ensures that a4 != 0. Alternatively, if a4 = 0, then the Fresnel surface contains the 2-dimensional plane x1=x2=0.**

In[34]:=
```
fresnel = emKappaToFresnel[kappa, {x0, x1, x2, x3}];
FullSimplify[fresnel /. {a4 → 0}]
```

Out[35]= $-b1^3 \left(x1^2 + x2^2\right) (x0 - x3) (x0 + x3)$