In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
<< helper.m
```

```
Loading KappaLib v1.2

Loading helper.m..
```

- **Define Metaclass II with parameters:**

  alpha_i in R,   beta_i in R\0,   and   beta_i all have same sign.

In[4]:=
```
kappa = emMatrixToKappa[( a1  -b1  0   0   0   0
                          b1  a1   0   0   0   0
                          0   0    a2  0   0   -b2
                          0   1    0   a1  b1  0
                          1   0    0   -b1 a1  0
                          0   0    b2  0   0   a2 )];
```

Write out algebraic equations that kappa satisfies and eliminate variables for A and B

In[5]:=
```
eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
AA = emMatrix["A", 4, Structure → "AntiSymmetric"];
BB = emMatrix["B", 4, Structure → "AntiSymmetric"];
RHS = -lambda emIdentityKappa[] + emBiProduct[rho, AA, BB] + emBiProduct[rho, BB, AA];
```

- **Since rho, A,B are all non-zero, we may scale A and assume that rho = 1**

In[10]:=
```
rho = 1;
```

■ Definition of decomposable medium (with

In[11]:=
```
eqs = simp[Union[Flatten[LHS - RHS]]];
show[eqs]
```

Out[12]//MatrixForm=
```
 1  :  0
 2  :  4 A24 B24
 3  :  - 4 A34 B34
 4  :  2 (b1 - 2 A12 B12)
 5  :  2 (b1 + 2 A13 B13)
 6  :  2 (A14 B13 + A13 B14)
 7  :  2 (A23 B13 + A13 B23)
 8  :  2 (A24 B14 + A14 B24)
 9  :  2 (A24 B23 + A23 B24)
10  :  2 (A34 B24 + A24 B34)
11  :  - 2 (A14 B12 + A12 B14)
12  :  - 2 (A14 B13 + A13 B14)
13  :  - 2 (A23 B12 + A12 B23)
14  :  - 2 (A23 B13 + A13 B23)
15  :  - 2 (A24 B14 + A14 B24)
16  :  - 2 (A24 B23 + A23 B24)
17  :  - 2 (A34 B14 + A14 B34)
18  :  - 2 (A34 B23 + A23 B34)
19  :  - 2 (A34 B24 + A24 B34)
20  :  - 4 A23 B23 - 2 b2 (a2 + mu)
21  :  - 4 A14 B14 + 2 b2 (a2 + mu)
22  :  2 (a1 - A13 B12 - A12 B13 + mu)
23  :  - 2 (a1 - A13 B12 - A12 B13 + mu)
24  :  2 (A24 B12 + A12 B24 + b1 (a1 + mu))
25  :  2 (A34 B13 + A13 B34 + b1 (a1 + mu))
26  :  - 2 (A24 B12 + A12 B24 + b1 (a1 + mu))
27  :  - 2 (A34 B13 + A13 B34 + b1 (a1 + mu))
28  :  - b1^2 + 2 A24 B13 + 2 A13 B24 + lambda + (a1 + mu)^2
29  :  - b1^2 - 2 A34 B12 - 2 A12 B34 + lambda + (a1 + mu)^2
30  :  - 2 A23 B14 - b2^2 - 2 A14 B23 + lambda + (a2 + mu)^2
```

In[13]:=
```
elimVars = Join[Variables[AA], Variables[BB]]
```

Out[13]=
```
{A12, A13, A14, A23, A24, A34, B12, B13, B14, B23, B24, B34}
```

In[14]:=
```
condVars = Join[Variables[kappa], {lambda, mu}]
```

Out[14]=
```
{a1, a2, b1, b2, lambda, mu}
```

■ Eliminate variables using a Gröbner basis

In[15]:=
```
gb = GroebnerBasis[eqs, condVars, elimVars]; // Timing
gb = simp[gb]; // Timing
Length[gb]
```

Out[15]=
```
{85.6476, Null}
```

Out[16]=
```
{0.739262, Null}
```

Out[17]=
```
31
```

In[18]:=
```
show[gb]
```

Out[18]//MatrixForm=

```
1  :  b1^4 b2 (a2 + mu)
2  :  b2 lambda^2 (a2 + mu)
3  :  b1 b2 lambda (a2 + mu)
4  :  b1^4 b2 (b2^2 - lambda)
5  :  b2 (b2^2 - lambda) lambda^2
6  :  lambda (b1^3 - b1 lambda)^3
7  :  b1^3 b2 (a1 + mu) (a2 + mu)
8  :  b1 b2 (b2^2 - lambda) lambda
9  :  b2 lambda (a1 + mu) (a2 + mu)
10 :  b1^3 b2 (b2^2 - lambda) (a1 + mu)
11 :  b2 (a2 + mu) (b1^2 + (a1 + mu)^2)
12 :  b2 (b2^2 - lambda) lambda (a1 + mu)
13 :  b1^4 (-b2^2 + lambda + (a2 + mu)^2)
14 :  b1^3 (b1^2 - lambda) lambda (a1 + mu)
15 :  b2 (b2^2 - lambda) (b1^2 + (a1 + mu)^2)
16 :  lambda^2 (-b2^2 + lambda + (a2 + mu)^2)
17 :  b1 lambda (-b2^2 + lambda + (a2 + mu)^2)
18 :  b1^3 (a1 + mu) (b1^2 - lambda + (a1 + mu)^2)
19 :  b1^3 (a1 + mu) (-b2^2 + lambda + (a2 + mu)^2)
20 :  lambda (a1 + mu) (-b2^2 + lambda + (a2 + mu)^2)
21 :  b1 lambda (a1 + mu) (-b1^2 + lambda + (a1 + mu)^2)
22 :  (b1^2 + (a1 + mu)^2) (-b2^2 + lambda + (a2 + mu)^2)
23 :  b1 (b2^2 - lambda + (a2 + mu)^2) (b2^2 + lambda + (a2 + mu)
24 :  lambda (b2^2 - lambda + (a2 + mu)^2) (b2^2 + lambda + (a2 + m
25 :  (a1 + mu) (b2^2 - lambda + (a2 + mu)^2) (b2^2 + lambda + (a2 +
26 :  b1 lambda (-5 b1^6 + 14 b1^4 lambda - 13 b1^2 lambda^2 + 4 lambda^2 (lambd
27 :  b1 (b1^2 - lambda) ((b1^2 - lambda)^2 + a1^2 (b1^2 + lambda) + 2 a1 (b1^2 + lambda)
28 :  -b1 lambda ((b1^2 - lambda)^2 + a1^2 (-5 b1^2 + lambda) + 2 a1 (-5 b1^2 + lambda) mu
29 :  lambda (a1^2 - b1^2 + lambda - 2 b1 mu + mu^2 + 2 a1 (-b1 + mu)) (a1^2 - b1^2 + lambda + 2
30 :  b1 (3 a1^4 - (b1^2 - lambda)^2 + 12 a1^3 mu + 2 (b1^2 + lambda) mu^2 + 3 mu^4 + 4 a1 mu (b1^2 + lambda -
31 :  (a1 + mu) (a1^4 - 3 b1^4 + 4 a1^3 mu + 2 b1^2 (lambda - mu^2) + (lambda + mu^2)^2 + 4 a1 mu (-b1^2 + lambd
```

- **Equation (1) implies that mu = -a2**
  Equation (4) implies that lambda = b2^2

In[19]:=
```
subs = {mu → -a2, lambda → b2^2};
```

In[20]:= **show[simp[gb /. subs]]**

Out[20]//MatrixForm=

$$\begin{pmatrix} 1 & : & 0 \\ 2 & : & b2^2 \left(b1^3 - b1\, b2^2\right)^3 \\ 3 & : & (a1 - a2)\, b1^3\, (b1 - b2)\, b2^2\, (b1 + b2) \\ 4 & : & (a1 - a2)\, b1^3 \left((a1 - a2)^2 + b1^2 - b2^2\right) \\ 5 & : & (a1 - a2)\, b1\, b2^2 \left((a1 - a2)^2 - b1^2 + b2^2\right) \\ 6 & : & -b1\, b2^2 \left(-5\, (a1 - a2)^2\, b1^2 + b1^4 + \left((a1 - a2)^2 - 2\, b1^2\right) b2^2 + b2^4\right) \\ 7 & : & b1\, b2^2 \left(-5\, b1^6 + 14\, b1^4\, b2^2 - 13\, b1^2\, b2^4 + 4\, b2^4 \left((a1 - a2)^2 + b2^2\right)\right) \\ 8 & : & b1\, (b1 - b2)\, (b1 + b2) \left((a1 - a2)^2\, b1^2 + b1^4 + \left((a1 - a2)^2 - 2\, b1^2\right) b2^2 + b2 \right. \\ 9 & : & b2^2 \left((a1 - a2)^2 + 2\, (a1 - a2)\, b1 - b1^2 + b2^2\right) \left(a1^2 + a2^2 + 2\, a2\, b1 - b1^2 - 2\, a1\, (a2 + \right. \\ 10 & : & b1 \left(3\, a1^4 - 12\, a1^3\, a2 + 3\, a2^4 - \left(b1^2 - b2^2\right)^2 + 2\, a2^2 \left(b1^2 + b2^2\right) - 4\, a1\, a2 \left(3\, a2^2 + b1^2 + b2^2\right) + 2\, a \right. \\ 11 & : & (a1 - a2) \left(a1^4 - 4\, a1^3\, a2 - 3\, b1^4 + 2\, b1^2 \left(-a2^2 + b2^2\right) + \left(a2^2 + b2^2\right)^2 - 4\, a1\, a2 \left(a2^2 - b1^2 + b2^2\right) + 2 \right. \end{pmatrix}$$

- **Since b1, b2 have the same sign, equation (2) implies that b1 = b2.**

In[21]:= **subs = Append[subs, b2 → b1]**

Out[21]= $\{mu \to -a2,\ lambda \to b2^2,\ b2 \to b1\}$

In[22]:= **show[simp[gb //. subs]]**

Out[22]//MatrixForm=

$$\begin{pmatrix} 1 & : & 0 \\ 2 & : & (a1 - a2)^5 \\ 3 & : & (a1 - a2)^3\, b1^3 \\ 4 & : & 4\, (a1 - a2)^2\, b1^5 \\ 5 & : & 4\, (a1 - a2)^2\, b1^7 \\ 6 & : & 3\, (a1 - a2)^4\, b1 + 4\, (a1 - a2)^2\, b1^3 \\ 7 & : & (a1 - a2)^4\, b1^2 - 4\, (a1 - a2)^2\, b1^4 \end{pmatrix}$$

- **Equation (2) implies that a2=a1**

In[23]:= **subs = Append[subs, a2 → a1]**

Out[23]= $\{mu \to -a2,\ lambda \to b2^2,\ b2 \to b1,\ a2 \to a1\}$

In[24]:= **show[simp[gb //. subs]]**

Out[24]//MatrixForm=

$$\begin{pmatrix} 1 & : & 0 \end{pmatrix}$$

In[25]:= **subs**

Out[25]= $\{mu \to -a2,\ lambda \to b2^2,\ b2 \to b1,\ a2 \to a1\}$