In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
<< helper.m
```

```
Loading KappaLib v1.2

Loading helper.m..
```

- **Define Metaclass III with parameters:**

  alpha_i in R, beta_i in R\0, and beta_i all have same sign.

In[4]:=
```
kappa = emMatrixToKappa[{{a1, -b1, 0, 0, 0, 0}, {b1, a1, 0, 0, 0, 0}, {1, 0, a1, 0, 0, -b1}, {0, 0, 0, a1, b1, 1}, {0, 0, 1, -b1, a1, 0}, {0, 1, b1, 0, 0, a1}}];
```

Write out algebraic equations that kappa satisfies and eliminate variables for A and B

In[5]:=
```
eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
AA = emMatrix["A", 4, Structure → "AntiSymmetric"];
BB = emMatrix["B", 4, Structure → "AntiSymmetric"];
RHS = - lambda emIdentityKappa[] + emBiProduct[rho, AA, BB] + emBiProduct[rho, BB, AA];
```

- **Since rho, A,B are all non-zero, we may scale A and assume that rho = 1**

In[10]:=
```
rho = 1;
```

In[11]:=
```
eqs = simp[Union[Flatten[LHS - RHS]]];
show[eqs]
```

Out[12]//MatrixForm=

1	:	0
2	:	4 A13 B13
3	:	4 A24 B24
4	:	-4 A12 B12
5	:	-4 A34 B34
6	:	2 (A24 B14 + A14 B24)
7	:	2 (A24 B23 + A23 B24)
8	:	2 (A34 B24 + A24 B34)
9	:	-2 (A24 B14 + A14 B24)
10	:	-2 (A24 B23 + A23 B24)
11	:	-2 (A34 B14 + A14 B34)
12	:	-2 (A34 B23 + A23 B34)
13	:	-2 (A34 B24 + A24 B34)
14	:	1 - 2 A13 B12 - 2 A12 B13
15	:	-1 + 2 A13 B12 + 2 A12 B13
16	:	2 (b1 + A23 B13 + A13 B23)
17	:	-2 (b1 + A23 B13 + A13 B23)
18	:	-4 A23 B23 - 2 b1 (a1 + mu)
19	:	-4 A14 B14 + 2 b1 (a1 + mu)
20	:	-2 (-b1 + A14 B12 + A12 B14)
21	:	2 (a1 - A14 B13 - A13 B14 + mu)
22	:	2 (a1 - A23 B12 - A12 B23 + mu)
23	:	-2 (a1 - A14 B13 - A13 B14 + mu)
24	:	2 (A24 B12 + A12 B24 + b1 (a1 + mu))
25	:	2 (A34 B13 + A13 B34 + b1 (a1 + mu))
26	:	-2 (A24 B12 + A12 B24 + b1 (a1 + mu))
27	:	-2 (A34 B13 + A13 B34 + b1 (a1 + mu))
28	:	$-b1^2 - 2\,A23\,B14 - 2\,A14\,B23 + lambda + (a1 + mu)^2$
29	:	$-b1^2 + 2\,A24\,B13 + 2\,A13\,B24 + lambda + (a1 + mu)^2$
30	:	$-b1^2 - 2\,A34\,B12 - 2\,A12\,B34 + lambda + (a1 + mu)^2$

In[13]:=
```
elimVars = Join[Variables[AA], Variables[BB]]
```

Out[13]= {A12, A13, A14, A23, A24, A34, B12, B13, B14, B23, B24, B34}

In[14]:=
```
condVars = Join[Variables[kappa], {lambda, mu}]
```

Out[14]= {a1, b1, lambda, mu}

- **Eliminate variables using a Gröbner basis**

In[15]:=
```
gb = GroebnerBasis[eqs, condVars, elimVars]; // Timing
gb = simp[gb]; // Timing
Length[gb]
```

Out[15]= {1.18772, Null}

Out[16]= {0.022768, Null}

Out[17]= 6

In[18]:=
```
show[gb]
```

Out[18]//MatrixForm=

1	:	$b1^3$
2	:	$lambda^2$
3	:	b1 lambda
4	:	b1 (a1 + mu)
5	:	lambda (a1 + mu)
6	:	$-3\,b1^2 - lambda + 3\,(a1 + mu)^2$

- **Equation (1) implies the contradiction b1 = 0. Hence Metaclass III is not possible.**