

```
In[1]:= SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
<< helper.m

Loading KappaLib v1.2
Loading helper.m..
```

■ Define Metaclass VII with parameters:

alpha_i in R, beta_i in R\0, and beta_i all have same sign.

```
In[4]:= kappa = emMatrixToKappa[ $\begin{pmatrix} a_1 & 0 & 0 & a_4 & 0 & 0 \\ 0 & a_2 & 0 & 0 & a_5 & 0 \\ 0 & 0 & a_3 & 0 & 0 & a_6 \\ a_4 & 0 & 0 & a_1 & 0 & 0 \\ 0 & a_5 & 0 & 0 & a_2 & 0 \\ 0 & 0 & a_6 & 0 & 0 & a_3 \end{pmatrix}$ ];
```

Write out algebraic equations that kappa satisfies and eliminate variables for A and B

```
In[5]:= eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
AA = emMatrix["A", 4, Structure -> "AntiSymmetric"];
BB = emMatrix["B", 4, Structure -> "AntiSymmetric"];
RHS = -lambda emIdentityKappa[] + emBiProduct[rho, AA, BB] + emBiProduct[rho, BB, AA];
```

■ Since rho, A,B are all non-zero, we may scale A and assume that rho = 1

```
In[10]:= rho = 1;
```

```
In[11]:= eqs = simp[Union[Flatten[LHS - RHS]]];
show[eqs]
Out[12]//MatrixForm=
```

1 :	0
2 :	2 (A13 B12 + A12 B13)
3 :	2 (A14 B13 + A13 B14)
4 :	2 (A23 B13 + A13 B23)
5 :	2 (A24 B12 + A12 B24)
6 :	2 (A24 B14 + A14 B24)
7 :	2 (A24 B23 + A23 B24)
8 :	2 (A34 B13 + A13 B34)
9 :	2 (A34 B24 + A24 B34)
10 :	- 2 (A13 B12 + A12 B13)
11 :	- 2 (A14 B12 + A12 B14)
12 :	- 2 (A14 B13 + A13 B14)
13 :	- 2 (A23 B12 + A12 B23)
14 :	- 2 (A23 B13 + A13 B23)
15 :	- 2 (A24 B12 + A12 B24)
16 :	- 2 (A24 B14 + A14 B24)
17 :	- 2 (A24 B23 + A23 B24)
18 :	- 2 (A34 B13 + A13 B34)
19 :	- 2 (A34 B14 + A14 B34)
20 :	- 2 (A34 B23 + A23 B34)
21 :	- 2 (A34 B24 + A24 B34)
22 :	4 A13 B13 - 2 a5 (a2 + mu)
23 :	4 A24 B24 - 2 a5 (a2 + mu)
24 :	- 4 A12 B12 + 2 a4 (a1 + mu)
25 :	- 4 A34 B34 + 2 a4 (a1 + mu)
26 :	- 4 A14 B14 + 2 a6 (a3 + mu)
27 :	- 4 A23 B23 + 2 a6 (a3 + mu)
28 :	a4 ² - 2 A34 B12 - 2 A12 B34 + lambda + (a1 + mu) ²
29 :	a5 ² + 2 A24 B13 + 2 A13 B24 + lambda + (a2 + mu) ²
30 :	a6 ² - 2 A23 B14 - 2 A14 B23 + lambda + (a3 + mu) ²

```
In[13]:= elimVars = Join[Variables[AA], Variables[BB]]
Out[13]= {A12, A13, A14, A23, A24, A34, B12, B13, B14, B23, B24, B34}
In[14]:= condVars = Join[Variables[kappa], {lambda, mu}]
Out[14]= {a1, a2, a3, a4, a5, a6, lambda, mu}
```

■ Eliminate variables using a Gröbner basis

```
In[15]:= gb = GroebnerBasis[eqs, condVars, elimVars]; // Timing
gb = simp[gb]; // Timing
Length[gb]
Out[15]= {68.7657, Null}
Out[16]= {1.13145, Null}
Out[17]= 45
```

In[18]:= **show[gb]**

Out[18]//MatrixForm=

1 :	$a_4 a_5 a_6 (a_1 + \mu) (a_2 + \mu) (a_3 + \mu)$
2 :	$a_4 a_5 a_6 (a_6^2 + \lambda) (a_1 + \mu) (a_2 + \mu)$
3 :	$a_4 a_5 a_6 (a_5^2 + \lambda) (a_1 + \mu) (a_3 + \mu)$
4 :	$a_4 a_5 a_6 (a_4^2 + \lambda) (a_2 + \mu) (a_3 + \mu)$
5 :	$a_4 a_5 a_6 (a_5^2 + \lambda) (a_6^2 + \lambda) (a_1 + \mu)$
6 :	$a_4 a_5 a_6 (a_4^2 + \lambda) (a_6^2 + \lambda) (a_2 + \mu)$
7 :	$a_4 a_5 a_6 (a_4^2 + \lambda) (a_5^2 + \lambda) (a_3 + \mu)$
8 :	$a_5 a_6 (a_2 + \mu) (a_3 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2)$
9 :	$a_4 a_6 (a_1 + \mu) (a_3 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2)$
10 :	$a_4 a_5 (a_1 + \mu) (a_2 + \mu) (a_6^2 + \lambda + (a_3 + \mu)^2)$
11 :	$a_4 a_5 a_6 (a_4^2 + \lambda) (a_5^2 + \lambda) (a_6^2 + \lambda)$
12 :	$a_5 a_6 (a_6^2 + \lambda) (a_2 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2)$
13 :	$a_5 a_6 (a_5^2 + \lambda) (a_3 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2)$
14 :	$a_4 a_6 (a_6^2 + \lambda) (a_1 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2)$
15 :	$a_4 a_6 (a_4^2 + \lambda) (a_3 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2)$
16 :	$a_4 a_5 (a_5^2 + \lambda) (a_1 + \mu) (a_6^2 + \lambda + (a_3 + \mu)^2)$
17 :	$a_4 a_5 (a_4^2 + \lambda) (a_2 + \mu) (a_6^2 + \lambda + (a_3 + \mu)^2)$
18 :	$a_5 a_6 (a_5^2 + \lambda) (a_6^2 + \lambda) (a_4^2 + \lambda + (a_1 + \mu)^2)$
19 :	$a_4 a_6 (a_4^2 + \lambda) (a_6^2 + \lambda) (a_5^2 + \lambda + (a_2 + \mu)^2)$
20 :	$a_4 a_5 (a_4^2 + \lambda) (a_5^2 + \lambda) (a_6^2 + \lambda + (a_3 + \mu)^2)$
21 :	$a_5 (a_2 + \mu) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2)$
22 :	$a_6 (a_3 + \mu) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2)$
23 :	$a_4 (a_1 + \mu) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2)$
24 :	$a_6 (a_3 + \mu) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2)$
25 :	$a_4 (a_1 + \mu) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2)$
26 :	$a_5 (a_2 + \mu) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2)$
27 :	$a_6 (a_3 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_5^2 + \lambda + (a_2 + \mu)^2)$
28 :	$a_5 (a_2 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2)$
29 :	$a_4 (a_1 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2)$
30 :	$a_5 (a_5^2 + \lambda) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2)$
31 :	$a_6 (a_6^2 + \lambda) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2)$
32 :	$a_4 (a_4^2 + \lambda) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2)$
33 :	$a_6 (a_6^2 + \lambda) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2)$
34 :	$a_4 (a_4^2 + \lambda) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2)$
35 :	$a_5 (a_5^2 + \lambda) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2)$
36 :	$a_6 (a_6^2 + \lambda) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_5^2 + \lambda + (a_2 + \mu)^2)$
37 :	$a_5 (a_5^2 + \lambda) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2)$
38 :	$a_4 (a_4^2 + \lambda) (a_5^2 + \lambda + (a_2 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2)$
39 :	$(a_5^2 + \lambda + (a_2 + \mu)^2) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2)$
40 :	$(a_6^2 + \lambda + (a_3 + \mu)^2) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2)$
41 :	$(a_4^2 + \lambda + (a_1 + \mu)^2) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2)$
42 :	$(a_6^2 + \lambda + (a_3 + \mu)^2) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2)$
43 :	$(a_4^2 + \lambda + (a_1 + \mu)^2) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2)$
44 :	$(a_5^2 + \lambda + (a_2 + \mu)^2) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2)$
45 :	$(a_4^2 + \lambda + (a_1 + \mu)^2) (a_5^2 + \lambda + (a_2 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2)$

■ Equation $gb[[45]] = 0$ contradicts $\lambda > 0$.

```
In[19]:= show[{gb[[45]]}]
```

Out[19]//MatrixForm=

$$(1 : (a4^2 + \lambda + (a1 + \mu)^2) (a5^2 + \lambda + (a2 + \mu)^2) (a6^2 + \lambda + (a3 + \mu)^2))$$

■ Thus κ can not be in Metaclass VII