```
In[1]:= SetDirectory["~/writing/WIP/KappaLib/"];
        << kappaLib.m
        << helper.m

        KappaLib v1.1

        Loading helper.m..

In[4]:= vars = {x0, x1, x2, x3};
```

Factorisation of Metaclass I

In[5]:= kappa = emMatrixToKappa

$$\left[\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & -b3 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & b3 & 0 & 0 & a3 \end{pmatrix}\right];$$

```
In[6]:= subs = {a3 → a2, b3 → b2};
        kappa = kappa //. subs;
        FullSimplify[emDet[kappa]]
```

Out[8]= $(a1^2 + b1^2)\,(a2^2 + b2^2)^2$

```
In[9]:= fresnel = emKappaToFresnel[kappa, vars];
        FullSimplify[fresnel]
```

Out[10]= $b2\,\left(-b1^2\,(x0 - x1)\,(x0 + x1)\,(x2^2 + x3^2) - \left((a1 - a2)^2 + b2^2\right)\,(x0 - x1)\,(x0 + x1)\,(x2^2 + x3^2) + b1\,b2\,\left((x0^2 - x1^2)^2 + (x2^2 + x3^2)^2\right)\right)$

In[11]:= $DD = \dfrac{(a1 - a2)^2 + b1^2 + b2^2}{b1\,b2};$

- **Since a1!= a2 or b1!= b2 (or both) we have DD > 2. Hence**

$$\frac{1}{2}\left(-DD + \sqrt{-4 + DD^2}\right) < 0, \qquad \frac{1}{2}\left(-DD - \sqrt{-4 + DD^2}\right) < 0$$

and the below matrices both have Lorentz signature

In[13]:= $AA = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\left(-DD + \sqrt{-4 + DD^2}\right) & 0 \\ 0 & 0 & 0 & \frac{1}{2}\left(-DD + \sqrt{-4 + DD^2}\right) \end{pmatrix};$

$BB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\left(-DD - \sqrt{-4 + DD^2}\right) & 0 \\ 0 & 0 & 0 & \frac{1}{2}\left(-DD - \sqrt{-4 + DD^2}\right) \end{pmatrix};$

```
In[15]:= Simplify[fresnel - b1 b2 b2 (vars.AA.vars) (vars.BB.vars)]

Out[15]= 0
```

Factorisation of Metaclass II

In[17]:=
```
kappa = emMatrixToKappa[( a1  -b1  0   0    0   0
                          b1   a1  0   0    0   0
                          0    0   a2  0    0  -b2
                          0    1   0   a1   b1  0
                          1    0   0  -b1   a1  0
                          0    0   b2  0    0   a2 )];

subs = {a2 → a1, b2 → b1};
kappa = kappa //. subs;
FullSimplify[emDet[kappa]]
```

Out[20]= $\left(a1^2+b1^2\right)^3$

In[21]:=
```
fresnel = emKappaToFresnel[kappa, vars];
FullSimplify[fresnel]
```

Out[22]= $-b1\ x0^4+b1^3\left(x1^2+x2^2-2\ x0\ x3\right)^2$

In[23]:=
```
AA = ( 1   0    0   b1
       0  -b1   0   0
       0   0   -b1  0
       b1  0    0   0 );

BB = ( -1  0    0   b1
       0  -b1   0   0
       0   0   -b1  0
       b1  0    0   0 );

Simplify[fresnel - b1 (vars.AA.vars) (vars.(BB).vars)]
```

Out[25]= 0

- **Since**

  Det(AA) <0,
  Det(BB)<0,

  matrices AA and BB both have Lorentz signatures.

In[26]:=
```
Det[AA]
Det[BB]
```

Out[26]= $-b1^4$

Out[27]= $-b1^4$

Factorisation of Metaclass IV

In[28]:=
```
kappa = emMatrixToKappa[( a1  0   0  -b1  0   0
                          0   a2  0   0  -b2  0
                          0   0   a3  0   0   a4
                          b1  0   0   a1  0   0
                          0   b2  0   0   a2  0
                          0   0   a4  0   0   a3 )];

subs = {a2 → a1, b2 → b1};
kappa = kappa //. subs;
FullSimplify[emDet[kappa]]
```

Out[31]= $(a3-a4)\ (a3+a4)\ \left(a1^2+b1^2\right)^2$

In[32]:=
```
fresnel = emKappaToFresnel[kappa, vars];
FullSimplify[fresnel]
```

Out[33]=

$$b1\left(a4^2\left(x1^2+x2^2\right)(x0-x3)(x0+x3)-\left((a1-a3)^2+b1^2\right)\left(x1^2+x2^2\right)(x0-x3)(x0+x3)+a4\,b1\left(-\left(x1^2+x2^2\right)^2+\left(x0^2-x3^2\right)^2\right)\right)$$

In[34]:=

$$D1=\frac{(a1-a3)^2-a4^2+b1^2}{a4\,b1};$$

$$AA=\begin{pmatrix}1&0&0&0\\0&\frac{1}{2}\left(-D1+\sqrt{4+D1^2}\right)&0&0\\0&0&\frac{1}{2}\left(-D1+\sqrt{4+D1^2}\right)&0\\0&0&0&-1\end{pmatrix};$$

$$BB=\begin{pmatrix}1&0&0&0\\0&\frac{1}{2}\left(-D1-\sqrt{4+D1^2}\right)&0&0\\0&0&\frac{1}{2}\left(-D1-\sqrt{4+D1^2}\right)&0\\0&0&0&-1\end{pmatrix};$$

```
Simplify[fresnel - b1^2 a4 (vars.AA.vars) (vars.(BB).vars)]
```

Out[37]= 0

- **Note: We know that a4!=0 and b1!=0, so D1 is well defined. However, we have no inequality for D1 as in Metaclass I. Nevertheless**

$$\frac{1}{2}\left(-D1+\sqrt{4+D1^2}\right)>0,$$
$$\frac{1}{2}\left(-D1-\sqrt{4+D1^2}\right)<0.$$

Thus the above matrices both have Lorentz signature.