In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.1.m
```

```
KappaLib v1.1
```

# Metaclass I

In[3]:=
```
kappa = emMatrixToKappa[{{a1, 0, 0, -b1, 0, 0}, {0, a2, 0, 0, -b2, 0}, {0, 0, a3, 0, 0, -b3}, {b1, 0, 0, a1, 0, 0}, {0, b2, 0, 0, a2, 0}, {0, 0, b3, 0, 0, a3}}];
```

# We can exclude the case a1=a2=a3, b1=b2=b3

In[4]:=
```
fr = emKappaToFresnel[kappa, {x0, x1, x2, x3}] //. {a3 → a1, a2 → a1, b3 → b1, b2 → b1};
FullSimplify[fr]
```

Out[5]=
$$b1^3 \left(-x0^2 + x1^2 + x2^2 + x3^2\right)^2$$

# Define D1,D2,D3 as functions of kappa

In[6]:=
```
D1[kappa_] := Module[
    {m, a1, a2, a3, b1, b2, b3},
    m = emKappaToMatrix[kappa];
    a1 = m[[1]][[1]]; a2 = m[[2]][[2]]; a3 = m[[3]][[3]];
    b1 = m[[1]][[4]]; b2 = m[[2]][[5]]; b3 = m[[3]][[6]];
    Return[((a2 - a3)^2 + b2^2 + b3^2) / (b2 b3)];
   ];
D2[kappa_] := Module[
    {m, a1, a2, a3, b1, b2, b3},
    m = emKappaToMatrix[kappa];
    a1 = m[[1]][[1]]; a2 = m[[2]][[2]]; a3 = m[[3]][[3]];
    b1 = m[[1]][[4]]; b2 = m[[2]][[5]]; b3 = m[[3]][[6]];
    Return[((a1 - a3)^2 + b1^2 + b3^2) / (b1 b3)];

   ];
D3[kappa_] := Module[
    {m, a1, a2, a3, b1, b2, b3},
    m = emKappaToMatrix[kappa];
    a1 = m[[1]][[1]]; a2 = m[[2]][[2]]; a3 = m[[3]][[3]];
    b1 = m[[1]][[4]]; b2 = m[[2]][[5]]; b3 = m[[3]][[6]];
    Return[((a1 - a2)^2 + b1^2 + b2^2) / (b1 b2)];
   ];
```

In[9]:=
```
D1[kappa]
D2[kappa]
D3[kappa]
```

Out[9]=
$$\frac{(a2 - a3)^2 + b2^2 + b3^2}{b2\, b3}$$

Out[10]=
$$\frac{(a1 - a3)^2 + b1^2 + b3^2}{b1\, b3}$$

Out[11]=
$$\frac{(a1 - a2)^2 + b1^2 + b2^2}{b1\, b2}$$

## We can exchange D1 <-> D2 by an orientation preserving coordinate change

In[12]:=
```
J = {{1, 0, 0, 0}, {0, 0, -1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}};
Det[J]
kappaP = emCoordinateChange[kappa, J];
```

Out[13]= 1

In[15]:=
```
emKappaToMatrix[kappaP] // MatrixForm
```

Out[15]//MatrixForm=

$$\begin{pmatrix} a2 & 0 & 0 & -b2 & 0 & 0 \\ 0 & a1 & 0 & 0 & -b1 & 0 \\ 0 & 0 & a3 & 0 & 0 & -b3 \\ b2 & 0 & 0 & a2 & 0 & 0 \\ 0 & b1 & 0 & 0 & a1 & 0 \\ 0 & 0 & b3 & 0 & 0 & a3 \end{pmatrix}$$

In[16]:=
```
Simplify[D2[kappa] - D1[kappaP]]
Simplify[D1[kappa] - D2[kappaP]]
Simplify[D3[kappa] - D3[kappaP]]
```

Out[16]= 0

Out[17]= 0

Out[18]= 0

## We can exchange D2 <-> D3 by an orientation preserving coordinate change

In[19]:=
```
J = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, -1}, {0, 0, 1, 0}};
Det[J]
kappaP = emCoordinateChange[kappa, J];
```

Out[20]= 1

In[22]:=
```
emKappaToMatrix[kappaP] // MatrixForm
```

Out[22]//MatrixForm=

$$\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a3 & 0 & 0 & -b3 & 0 \\ 0 & 0 & a2 & 0 & 0 & -b2 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b3 & 0 & 0 & a3 & 0 \\ 0 & 0 & b2 & 0 & 0 & a2 \end{pmatrix}$$

In[23]:=
```
Simplify[D1[kappa] - D1[kappaP]]
Simplify[D2[kappa] - D3[kappaP]]
Simplify[D3[kappa] - D2[kappaP]]
```

Out[23]= 0

Out[24]= 0

Out[25]= 0