

- **Claim:** When Metaclass I factorises into a double light cone, the null cone intersection is two lines.

- **Define Lorentz null cones**

$$\text{In[1]:= } \mathbf{AA} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \left(-D3 + \sqrt{-4 + D3^2} \right) & 0 \\ 0 & 0 & 0 & \frac{1}{2} \left(-D3 + \sqrt{-4 + D3^2} \right) \end{pmatrix};$$

$$\mathbf{BB} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \left(-D3 - \sqrt{-4 + D3^2} \right) & 0 \\ 0 & 0 & 0 & \frac{1}{2} \left(-D3 - \sqrt{-4 + D3^2} \right) \end{pmatrix};$$

- **Suppose v is a null vector for both Lorentz metrics**

$$\text{In[3]:= } \mathbf{v} = \{a0, a1, a2, a3\}$$

$$\text{Out[3]= } \{a0, a1, a2, a3\}$$

$$\text{In[4]:= } \mathbf{p0} = \mathbf{v} \cdot \mathbf{AA} \cdot \mathbf{v};$$

$$\mathbf{p1} = \mathbf{v} \cdot \mathbf{BB} \cdot \mathbf{v};$$

- **If p0 = 0 and p1 = 0, then it follows that p0-p1 = 0.**

$$\text{In[6]:= } \mathbf{Simplify}[\mathbf{p0} - \mathbf{p1}]$$

$$\text{Out[6]= } (a2^2 + a3^2) \sqrt{-4 + D3^2}$$

- **It follows that a2=a3=0**

$$\text{In[7]:= } \mathbf{Simplify}[(\mathbf{p0} + \mathbf{p1}) /. \{a2 \rightarrow 0, a3 \rightarrow 0\}]$$

$$\text{Out[7]= } 2 (a0^2 - a1^2)$$

- **It follows that a0= +/- a1**

- **Check**

$$\text{In[8]:= } \mathbf{vec} = \{t, s, 0, 0\};$$

$$\mathbf{vec} \cdot \mathbf{AA} \cdot \mathbf{vec}$$

$$\mathbf{vec} \cdot \mathbf{BB} \cdot \mathbf{vec}$$

$$\text{Out[9]= } -s^2 + t^2$$

$$\text{Out[10]= } -s^2 + t^2$$