In[1]:=
```
SetDirectory["~/KappaLib/"];
<< kappaLib-1.1.m
<< helper.m
```

```
KappaLib v1.1

Loading helper.m..
```

In[4]:=
```
vp = {1.6385600128874258`, -2.79035110757227`, -0.9894755078507828`};
vv = {0.9919624506450033`, -0.11938437307145532`, -0.04192693617109755`};
```

In[5]:=
```
sub = {a3 → a2, b3 → b2};
```

In[6]:=
```
vars = {x0, x1, x2, x3};
```

$$\texttt{kappa = emMatrixToKappa}\left[\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & -b3 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & b3 & 0 & 0 & a3 \end{pmatrix}\right];$$

```
fr = emKappaToFresnel[kappa, vars] /. sub;
```

In[9]:=
```
FullSimplify[fr]
```

Out[9]=
$$b2\left(-b1^2 (x0-x1)(x0+x1)\left(x2^2+x3^2\right) - \left((a1-a2)^2+b2^2\right)(x0-x1)(x0+x1)\left(x2^2+x3^2\right) + b1\,b2\left(\left(x0^2-x1^2\right)^2+\left(x2^2+x3^2\right)^2\right)\right)$$

In[10]:=
$$\texttt{AA} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\left(-D3+\sqrt{-4+D3^2}\right) & 0 \\ 0 & 0 & 0 & \frac{1}{2}\left(-D3+\sqrt{-4+D3^2}\right) \end{pmatrix};$$

$$\texttt{BB} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\left(-D3-\sqrt{-4+D3^2}\right) & 0 \\ 0 & 0 & 0 & \frac{1}{2}\left(-D3-\sqrt{-4+D3^2}\right) \end{pmatrix};$$

$$\texttt{ss} = \left\{D3 \to \frac{\left((a1-a2)^2+b1^2+b2^2\right)}{b1\,b2}\right\};$$

In[13]:=
```
f1 = Simplify[vars.AA.vars];
f2 = Simplify[vars.BB.vars];
verify = b1 b2^2 f1 f2;
FullSimplify[( fr - verify) /. sub /. ss]
```

Out[16]= 0

- **Fresnel polynomial depends on x0, x1^2+x2^2, x3**

In[17]:=
```
f1
f2
```

Out[17]=
$$x0^2+\frac{1}{2}\left(-2\,x1^2+\left(-D3+\sqrt{-4+D3^2}\right)\left(x2^2+x3^2\right)\right)$$

Out[18]=
$$x0^2+\frac{1}{2}\left(-2\,x1^2-\left(D3+\sqrt{-4+D3^2}\right)\left(x2^2+x3^2\right)\right)$$

In[19]:=
$$\texttt{g1[x0\_, x1\_, x2\_, x3\_, D3\_] :=}\ x0^2+\frac{1}{2}\left(-2\,x1^2+\left(-D3+\sqrt{-4+D3^2}\right)\left(x2^2+x3^2\right)\right)$$

$$\texttt{g2[x0\_, x1\_, x2\_, x3\_, D3\_] :=}\ x0^2+\frac{1}{2}\left(-2\,x1^2-\left(D3+\sqrt{-4+D3^2}\right)\left(x2^2+x3^2\right)\right)$$

```
In[21]:= draw[d3_, xx_, yy_, zz_] := Module[
    {p1, p2, grayLevel},
    grayLevel = 0.2; (* 0 = black *)
    p1 = ContourPlot3D[
      {g1[x0, x1, 0, x3, d3] == 0},
      {x0, -xx, xx}, {x1, -yy, yy}, {x3, -zz, zz},
      Axes → False,
      Boxed → False,
      Lighting → "Neutral",
      ViewPoint → vp,
      Mesh → 5,
      PlotPoints → 40,
      ColorFunctionScaling → 0.1,
      MeshStyle → {Directive[GrayLevel[grayLevel], Opacity[0.5]]},
      ViewVertical → vv];
    p2 = ContourPlot3D[
      {g2[x0, x1, 0, x3, d3] == 0},
      {x0, 0, xx}, {x1, -yy, yy}, {x3, -zz, zz},
      MeshStyle → {Directive[GrayLevel[grayLevel], Opacity[0.5]]},
      Axes → False,
      Boxed → False,
      Mesh → {2, 2, 3},
      MaxRecursion → 10,
      Lighting → "Neutral",
      PlotPoints → 40,
      ViewPoint → vp,
      ViewVertical → vv];
    Show[{p1, p2}, PlotRange → {All, All, All}]
   ];

In[22]:= plot1 = draw[2.05, 1, 1, 2.165];
         plot2 = draw[2.3, 1, 1, 2.2];
         plot3 = draw[3, 1, 1, 2.9];
         ptot = Show[GraphicsGrid[{{plot1, plot2, plot3}}]];

In[26]:= Export["PlotI.pdf", ptot, ImageResolution → 1600]

Out[26]= PlotI.pdf
```

- **Note running the above notebook is relatively time consuming. Images have been removed from the above due their size.**