Timo Salin: *The quenching problem for the N-dimensional ball*; Helsinki University of Technology Institute of Mathematics Research Reports A459 (2003).

Abstract: *In this paper we consider the quenching problem for the reaction diffusion equation* $u_t - \Delta u = f(u)$ *with Cauchy-Dirichlet data, in the case where the reaction term is singular at* $u = 0$ *in the sense that* $\lim_{u \downarrow 0} f(u) = -\infty$. *For* $u > 0$ *we take* $f(u)$ *to be smooth and to satisfy* $(-1)^k f^{(k)}(u) < 0$, $k = 0, 1, 2$. *Furthermore we assume that the reaction term is weakly singular at the origin in the sense that:* $|u^n f^{(n)}(u)| = o(|f(u)|)$, *as* $u \downarrow 0$, $n = 1$ *and* $n = 2$.
We study the equation in $\Omega = \{x \in \mathcal{R}^N : |x| < R\}$ *and in the case where the initial function is radial, i.e.,* $u_0(x) = u_0(r)$ *and* $u_0'(r) \geq 0$ $(r = |x|)$. *We show that quenching occurs for sufficiently large* Ω *and prove the quenching rate estimate for the problem. We also study a refined asymptotic analysis of the solution near a quenching point for two examples of reaction terms. More precisely, we give an asymptotic expression for the solution with respect to space- and time variable in a backward space-time parabola near a quenching point.*

AMS subject classifications: 35K20,35K55,35K57

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