Ville Turunen: *Differentiability in locally compact metric spaces*; Helsinki University of Technology, Institute of Mathematics, Research Reports A487 (2005).

Abstract: *We study differentiability in locally compact metric spaces, using point derivations of Lipschitz functions. The classical Riemannian manifold case can be generalized to those locally compact metric spaces* $(M, (x, y) \mapsto |xy|)$ *for which there are local Cauchy–Schwarz -like inequalities*

$$|wx \cdot yz| \leq C\, |wx|\, |yz|,$$

where $C < \infty$ *is a constant and* $wx \cdot yz \in \mathbb{R}$ *is defined by*

$$wx \cdot yz := \frac{1}{2}\left(|wz|^2 + |xy|^2 - |wy|^2 - |xz|^2\right).$$

These Cauchy–Schwarz spaces *behave well with respect to elementary operations, e.g. under Gromov–Hausdorff limits. The* dot product $wx \cdot yz$ *can be thought as a discrete analogue of the Riemannian metric tensor. Examples of good and bad spaces in this respect are given. Moreover, we get new interpretations of concepts like Gromov hyperbolicity, comparison angles, Aleksandrov spaces of non-positive curvature and Reshetnyak's quadruple comparison.*

AMS subject classifications: 51K05, 53C45, 53C70, 54E40.

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