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Abstract: *We show how arbitrage can be generated by a change in volatility that is due to a change of stock exchange.*

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1 Introduction

In this note we study the change of the stock exchange from the perspective of the mathematical finance. In particular, we study option-pricing and arbitrage. We shall show that the change of stock exchange may make the already traded options mispriced, and this leads to arbitrage opportunities. Moreover, we give an explicit strategy illustrating how to benefit from this.

This study is motivated by the following example case: Recently US (Raleigh, NC) based software developer Red Hat, Inc. departed from NASDAQ Stock Exchange to be listed to New York Stock Exchange (NYSE). One of the stated intents of the relisting was to “reduce trading volatility” of the company stock [4]. The decision seems to be related to the standard Black-Scholes pricing model [2] used to determine the accounted cost of the stock options the company has granted.

In our setting we shall assume that the change of the stock exchange yields an automatic decrease in the volatility of the stock in question. In the last section, we show evidence that this assumption is feasible.

2 Setting

We assume that the discounted stock-price process follows the classical, non-homogeneous in volatility if stock exchange is changed, Black-Scholes model:

If no change of stock exchange is done then the discounted stock-price process $S_{\sigma_0}(t)$, $0 \le t \le T$, is given by the dynamics

$$\mathrm{d}S_{\sigma_0}(t) \;=\; \mu_0(t)S_{\sigma_0}(t)\,\mathrm{d}t + \sigma_0 S_{\sigma_0}(t)\,\mathrm{d}W(t), \qquad S_{\sigma_0}(0) = s_0, \tag{2.1}$$

where $\mu_0(t)$, $0 \le t \le T$, is the mean return function of the stock, the constant $\sigma_0 > 0$ is the volatility of the stock, and $W(t)$, $0 \le t \le T$, is a standard Brownian motion. If at some time $t_1 < T$ the stock is listed to a new stock exchange then the stock-price process $S_\sigma(t)$, $0 \le t \le T$, is given by the dynamics

$$\mathrm{d}S_\sigma(t) \;=\; \mu(t)S_\sigma(t)\,\mathrm{d}t + \sigma(t)S_\sigma(t)\,\mathrm{d}W(t), \qquad S_\sigma(0) = s_0, \tag{2.2}$$

where

$$\sigma(t) \;=\; \begin{cases} \sigma_0, & \text{if } t < t_1, \\ \sigma_1, & \text{if } t \ge t_1. \end{cases}$$

Motivated by the introduction we assume that $\sigma_1 < \sigma_0$. The mean function $\mu(t)$, $0 \le t \le T$, must of course satisfy $\mu(t) = \mu_0(t)$ for $t < t_1$.

The model described above is admittedly somewhat simplistic. However, the arbitrage we construct in the next section will hold in more complicated models. Next two remarks elaborate some possible generalizations to the models.

Remark 2.3 *It may not be reasonable to assume that the volatility σ_1 in the new stock exchange is deterministic. However, the claims of this note remain* *essentially true if one merely assumes that σ_1 is a $\mathcal{F}(t_1)$-measurable random variable, and $\sigma_1 < \sigma_0$ almost surely. Here $\mathcal{F}(t_1)$ is the σ-algebra generated by the stock-price process upto time t_1.*

Remark 2.4 *The classical Black-Scholes model assumes that the stock-price process is driven by a Brownian motion. In particular, this means that the log-returns are independent and Gaussian. There is, however, a lot of empirical evidence that the log-returns are neither independent nor Gaussian. Nevertheless, the results of this note remain essentially true if we consider a more general class of models where the log-returns are merely continuous, satisfying a certain small ball property, having the same volatility as the Brownian driven model. For details on these generalizations we refer to [1].*

Both models (2.1) and (2.2) fit well to the orthodoxy of Arbitrage Pricing Theory: They are free of arbitrage and complete (see e.g. [3] for details). There is a problem, however. The prices of the options in model (2.1) and (2.2) do not coincide, and this gives rise to arbitrage opportunities. Indeed, in the next section we construct one arbitrage opportunity by short-selling a convex European vanilla option on the stock.

3 Arbitrage

Let $f = f(S(T))$ be a European vanilla claim on the stock-price at the terminal date T. We assume that the function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is convex. So, e.g. call and put options are covered in our considerations.

If the stock exchange is not changed then we are in the domain of classical homogeneous Black-Scholes model. Indeed, suppose $S_{\sigma_0}(t) = x$. Then the standard martingale arguments together with Markovianity yield that the price of the option $f(S_{\sigma_0}(T))$ at the time $t < T$ is

$$\begin{aligned} v_{\sigma_0}(t,x) &= \mathbf{E}\Big[f\Big(xe^{\sigma_0(W(T)-W(t))-\frac{1}{2}\sigma_0^2(T-t)}\Big)\Big] \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f\Big(xe^{\sigma_0\sqrt{T-t}y-\frac{1}{2}\sigma_0^2(T-t)}\Big)\, e^{-\frac{1}{2}y^2}\,\mathrm{d}y \end{aligned} \tag{3.1}$$

(see e.g. [3] for details). Similarly, in the non-homogeneous case the price of the option $f(S_\sigma(T))$ at the time $t < t_1 < T$ is

$$\begin{aligned} v_{\sigma}(t,x) &= \mathbf{E}\Big[f\Big(xe^{\int_t^T\sigma(s)\mathrm{d}W(s)-\frac{1}{2}\int_t^T\sigma(s)^2\mathrm{d}s}\Big)\Big] \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f\Big(xe^{\sqrt{\int_t^T\sigma(s)^2\mathrm{d}s}\,y-\frac{1}{2}\int_t^T\sigma(s)^2\mathrm{d}s}\Big)\, e^{-\frac{1}{2}y^2}\,\mathrm{d}y, \end{aligned} \tag{3.2}$$

when $S_\sigma(t) = x$ (see e.g. [3] for details).

Now we show that for a convex option f the prices satisfy $v_{\sigma_0}(t,x) > v_\sigma(t,x)$ for all $x \in \mathbb{R}$ and $t < t_1$. This can be shown by using the formulas (3.1) and

(3.2) directly. We choose an easier path, however. We only show that $v_{\sigma_0}(t,x) > v_\sigma(t,x)$ holds for call options and the general claim for convex options follows then from the representation of a convex function as

$$f(x) \;=\; f(0) + f'(0)x + \int_0^\infty f''(y)(x-y)^+ \,\mathrm{d}y \tag{3.3}$$

(f' and f'' denote, if necessary, generalized derivatives). Indeed, equation (3.3) says that a convex claim f can be constructed by putting $f(0)$ amount of money in the money markets, buying $f'(0)$ shares of stock and for each $y > 0$ buying $f''(y)\mathrm{d}y$ number of call options.

Let us then consider the case of a call option. Using formulas (3.1) and (3.2), respectively, we see that the price functions of a call option with strike K are

$$v_{\sigma_0}^{\mathrm{call}}(t,x) \;=\; x\Phi\big(d_{\sigma_0}(t,x)\big) - K\Phi\Big(d_{\sigma_0}(t,x) - \sigma_0\sqrt{T-t}\Big), \tag{3.4}$$

$$v_{\sigma}^{\mathrm{call}}(t,x) \;=\; x\Phi\big(d_{\sigma}(t,x)\big) - K\Phi\left(d_{\sigma}(t,x) - \sqrt{\int_t^T \sigma(s)^2 \,\mathrm{d}s}\right), \tag{3.5}$$

where

$$\Phi(x) \;=\; \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}y^2}\,\mathrm{d}y$$

is the standard normal probability distribution function and

$$d_{\sigma_0}(t,x) \;=\; \frac{\ln\frac{x}{K} + \frac{1}{2}\sigma_0^2(T-t)}{\sigma_0\sqrt{T-t}},$$

$$d_{\sigma}(t,x) \;=\; \frac{\ln\frac{x}{K} + \frac{1}{2}\int_t^T \sigma(s)^2\mathrm{d}s}{\sqrt{\int_t^T \sigma(s)^2\mathrm{d}s}}.$$

But it is well-known that the function in the right-hand-side of (3.4) is strictly increasing in $\sigma_0^2(T-t)$. So, the claim $v_{\sigma_0}^{\mathrm{call}}(t,x) > v_{\sigma}^{\mathrm{call}}(t,x)$ follows from the fact that $\sigma_0^2(T-t) > \int_t^T \sigma(s)^2\mathrm{d}s$, since $\sigma_0 > \sigma_1$.

Now it is easy to see how to construct an arbitrage opportunity. Indeed, an informed investor who knows at the time 0 that the stock will be relisted at a future time $t_1 < T$ to a new stock exchange knows that the true model for the stock-price is (2.2). However, the “market in general” does not know this. It assumes that the true model is (2.1). Thus it prices options according to formula (3.1). But the informed investor knows that for convex options the price (3.1) is too high, and the option can be replicated with a lower price (3.2). So, the informed investor sells one convex claim short receiving $v_{\sigma_0}(0,s_0)$ amount of money. Then with capital $v_\sigma(0,s_0)$ she replicates the convex claim $f(S_\sigma(T))$ by using the standard Delta-hedging technique (see e.g. [3]). I.e. if $S_\sigma(t) = x$ she keeps

$$g_\sigma(t,x) \;=\; \frac{\partial}{\partial x} v_\sigma(t,x)$$

number of stocks and puts the remaining money

$$b_\sigma(t,x) \;=\; v_\sigma(t,x) - g_\sigma(t,x)\,x$$

in the discounted money market. Her riskless gain is the difference $v_{\sigma_0}(0, s_0) - v_\sigma(0, s_0) > 0$. So, the informed investor has made not only arbitrage, but strong arbitrage: She has generated strictly positive wealth with zero capital.

Remark 3.6 *If the new volatility σ_1 is not known but an $\mathcal{F}(t_1)$-measurable random variable then the arbitrage opportunity given above will still hold provided $\sigma_1(\omega) \leq \sigma_0 + \varepsilon$ for almost all ω. In this case the informed investor cannot hedge the claim f completely, but she can super-hedge it assuming that the new volatility is $\sigma_0 - \varepsilon$. So, the (strong) arbitrage opportunity remains.*

Example 3.7 *To further illustrate the arbitrage opportunity arising from changing a stock exchange let us consider a manager of a company who has a call option on the company's stock. The manager makes it so that the company's stock will change the stock exchange at a future date t_1. She knows that the future volatility σ_1 is smaller than the current volatility σ_0. Also, at time t_1 the price of her call option will decrease, at least in accounted value. Should the manager sell her call option immediately? Yes. She can replicate the call option with less money than she receives from selling it immediately. So, the decreased accounted value of the call option is transferred into an arbitrage opportunity for the manager. So, the old value of the the call option is equal to the new decreased value of the call option plus the arbitrage generated by following the strategy described above.*

The arbitrage opportunity constructed above was for an informed investor, i.e. for an insider. But the changing of a stock exchange admits, in principle, also arbitrage opportunities for the outsiders. Indeed, suppose that the company announces, as they usually do, at some time $t_0 < t_1$ that they will change the stock exchange at time t_1. Then the outsider will know from this "shock information" that the market price $v_{\sigma_0}(t_0, S_{\sigma_0}(t_0))$ at time t_0 for a convex claim f is too high and the correct replication price is $v_\sigma(t_0, S_\sigma(t_0))$. So, the newly informed investor can make arbitrage in a similar way as the informed insider investor does. Of course, if the markets are efficient, the price of the convex option f at time t_0 must decrease to its correct value $v_\sigma(t_0, S_\sigma(t_0))$ "instantaneously" so the outsider arbitrage opportunity vanishes from the markets. The insider arbitrage opportunities, however, remain.

4 Empirical Evidence

According to a press release issued on Nov. 17, 2006, Red Hat decided to switch from NASDAQ into the New York Stock Exchange, on their belief, that it would reduce trading volatility. The model (2.2) is constructed based on this essential assumption. To determine its feasibility, we present the empirical evidence below.

The failure to obtain sufficiently good data, from companies which had switched markets, has restricted our prospects to consider Red Hat stock prices as our unique reliable source. Also, we could not obtain any data about option prices on Red Hat's stock. We collected our data from the Datastream's global database at Helsinki School of Economics and Business Administration. We were limited to using the data of adjusted closing prices, i.e. revised prices to include any actions that occurred prior to the next day's open.

We calculated an annualized historical volatility of total returns with a window function of 60 points and 255 trading days (estimated number of trade days in a year). Mathematically,

$$\sigma_k^2 = \frac{1}{59}\sum_{j=0}^{59}\left(r_{k-j} - \frac{1}{60}\sum_{j=0}^{59} r_{k-j}\right)^2 \times 255,$$

where $r_j = \ln(S_{j+1}/S_j)$ and S_j is the stock price.

We used the so-called Bollinger bands [5] to identify periods of high and low volatility. Bollinger bands are a technical analysis trading tool introduced in the early 80's to adapt trading bands and the concept of volatility as a function of time, which it was believed to be static at the time. It is considered that prices are high at the upper band and low at the lower band. The Bollinger Bands consists of three curves designed to encompass the majority of a security's price dynamics. It is calculated according to equation (4.1). The middle band is a measure of the intermediate term consisting of a convolution with a window function of 20 adjusted closing prices and it serves as a base for the upper and lower bands. The width of the interval between the upper, the lower and the middle band is determined by the volatility. In this case, 2.5 times the standard deviation of the data used to calculate the middle band, the convolution:

$$BB_k^{\pm} = \frac{1}{20}\sum_{i=0}^{19} S_{k+i} \pm 2.5\sigma_k. \tag{4.1}$$

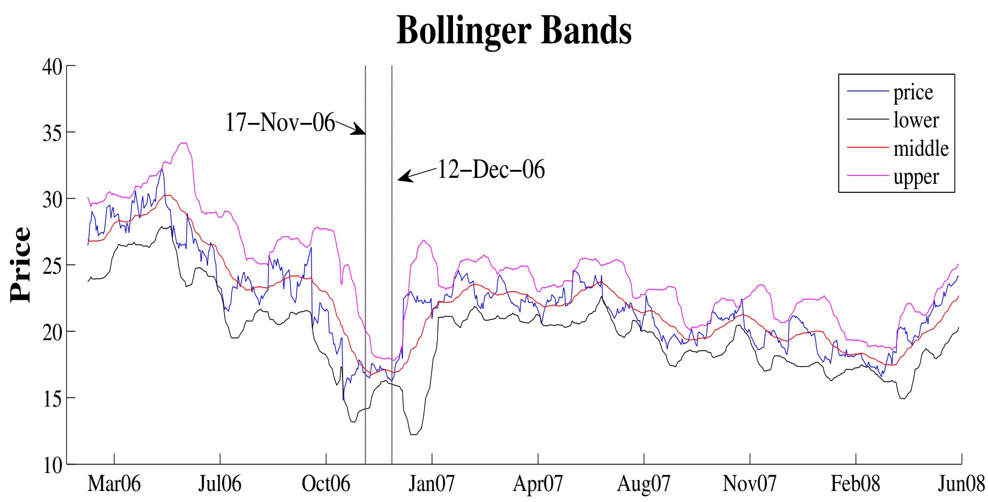

Figure 1: Bollinger bands along with the adjusted closing prices of Red Hat stock.

Figure 1 presents the Bollinger bands and the historical volatility of the total returns. The range of time is chosen from 28-Feb-06 until 30-May-08. The announcement day and the first day of trading at NYSE are shown by two vertical lines respectively. The price process exhibits less fluctuations and smoother signal after changing into the NYSE. This is translated into narrow Bollinger bands, a sign of stable lower volatility; in contrast with, the wider bands before the release press day; an indication of higher volatility. For the same reason, the second subplot shows that the historical volatility drops drastically after joining NYSE followed by a stable period lasting until now, the longest in Red Hat history, see Figure 2. However, it may be observed that the change of volatility is not immediate and even Bollinger bands became wider before getting narrow, or that in the historical volatility there is an intermediate interval of time before reaching the final level of volatility. This is due to the fast increase in price of the stock during the period immediately after switching the market.

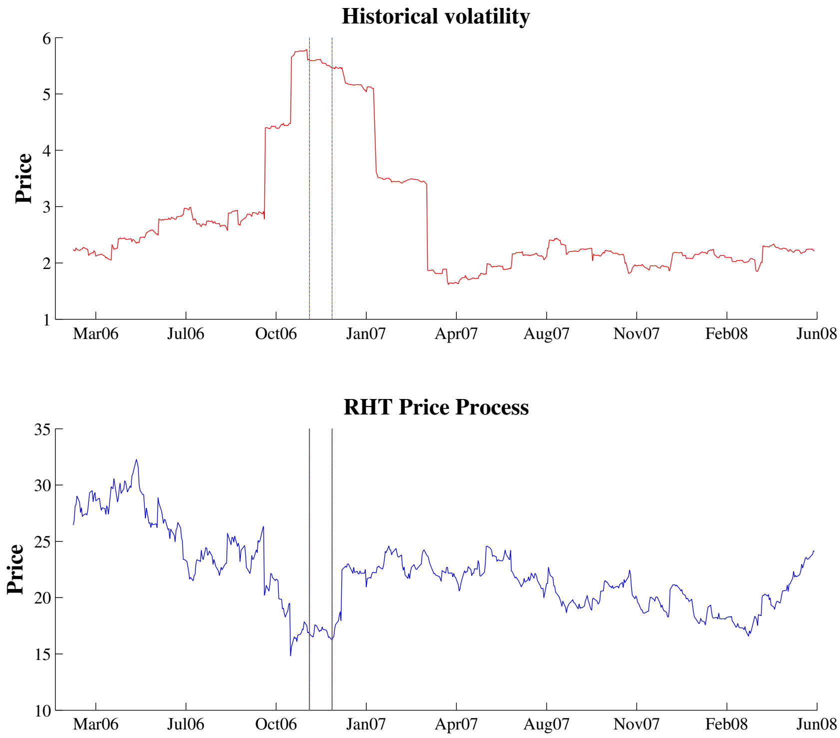

Figure 2: The historical volatility drops drastically after joining NYSE followed by a stable period lasting until now, the longest in Red Hat history.

We also carried out a left-tailed F-test according to [6]. Each set of data contains 368 realizations from both markets. We formulate the problem as follows: Consider σ_1 be the volatility of NYSE market and σ_2 be the one of NASDAQ market. We test $H_0 : \sigma_1 = \sigma_2$ against $H_1 : \sigma_1 < \sigma_2$ with a significance level of 1%.

The null hypothesis is rejection in favour of the alternative one with a p-value of 5.1×10^{-13} and a confidence interval of $[0, 0.5592]$ for the true ratio σ_1 to σ_2.

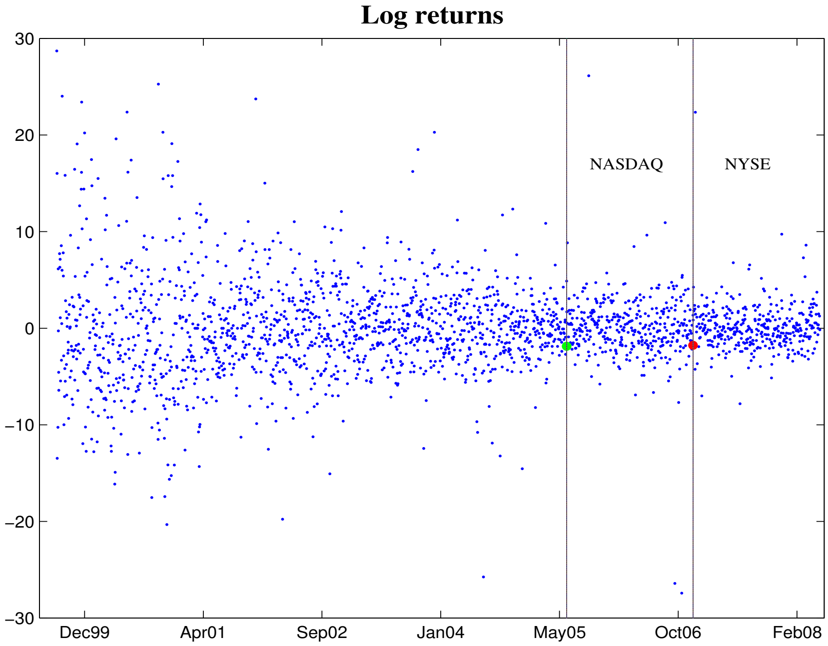

Figure 3: Sample sets are constructed from the most recient historical log returns with 368 units each one. The vertical lines shows the regions where each set belongs.

In short, the analysis confirms that the volatility has changed in a significant manner after switching the trading market and that the structural change in volatility described by the model (2.2) exits in a practical setting.

5 Conclusions

Options are sophisticated instruments. In the early days the options granted by the company were not accounted as expenses. Nowadays these contingent expenses are accounted by using the Black-Scholes paradigm. However, quite simple changes in market conditions can make the Black-Scholes paradigm unapplicable. In this note we showed that changing of the stock exchange is beyond the scope of the standard Black-Scholes pricing as the structural change in the volatility implies arbitrage. In the long run this unapplicability could lead to global unified stock exchange similar to FX-markets, fixing the problem. In the meanwhile one should be mindful of arbitrage opportunities.

References

[1] Bender, C., Sottinen, T., and Valkeila, E. (2008) Pricing by hedging and no-arbitrage beyond semimartingales. *Finance and Stochastics*, 12, 441-468.

[2] Black, F., and Scholes, M. (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* **81** (3): 637-654.

[3] Karatzas, I., and Shreve, S. (1998) *Methods of Mathematical Finance.* Springer.

[4] Red Hat Inc. press release Nov. 17, 2006. Available from `http://investors.redhat.com/phoenix.zhtml?c=67156&p=irol-newsArticle&ID=932665`

[5] Bollinger, J. (2002) *Bollinger on Bollinger Bands.* McGraw-Hill.

[6] Milton, J.S. and Arnold, J.C. (2002) *Introduction to Probability and Statistics.* McGraw-Hill.

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