Helsinki University of Technology Institute of Mathematics Reports

Espoo 2009 C021

WORKSHOP ON NON-SEMIMARTINGALE TECHNIQUES IN MATHEMATICAL FINANCE
HELSINKI UNIVERSITY OF TECHNOLOGY, MAY 26–28, 2009
ABSTRACTS

Ari-Pekka Perkkiö (ed.)

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TEKNISKA HÖGSKOLAN
HELSINKI UNIVERSITY OF TECHNOLOGY
TECHNISCHE UNIVERSITÄT HELSINKI
UNIVERSITE DE TECHNOLOGIE D'HELSINKI

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ABSTRACTS

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Department of Mathematics and Systems Analysis

Ari-Pekka Perkkiö (ed.): *Workshop on Non-Semimartingale Techniques in Mathematical Finance 2009 - Abstracts*; Helsinki University of Technology Institute of Mathematics Reports C021 (2009).

Abstract: *This report contains the program, list of participants and abstracts for the presentations of the international workshop on Non-Semimartingale Techniques in Mathematical Finance 2009, held at the Helsinki University of Technology, May 26–28, 2009.*

AMS subject classifications: 65-06

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1 General Information

Directions

Helsinki University of Technology (TKK) is located in Otaniemi, Espoo. The talks will be held in the lecture hall K which is located on the ground floor of the main building of TKK, next to the M-entrance.

Lunches

There is restaurant Alvari (Tuesday and Thursday lunches) on the ground floor of the main building. There is also restaurant Dipoli (Wednesday lunch) which is located in the campus. Tables are reserved for lunches which are free for workshop participants. Lunch tickets are handed out at the registration desk.

Computer access

Instructions about computers with passwords will be handed out at the registration desk. Please handle the password sheet responsibly. Wlan (Aalto open) is open access.

Social events

On Tuesday after the last talk the Get together will be held in the coffee room of the Institute of Mathematics.

On Wednesday there will be an opportunity to enjoy Finnish sauna. The sauna is a public Kotiharjun sauna close to the Helsinki city centre. A guide will leave from TKK at 17.30 and another guide from Hotel Arthur at 18.00. If you prefer to come to the sauna by your own, please be at the sauna around 18.30 and ask the organizers for the directions.

On Thursday the conference dinner will be at Restaurant Kuu at 19.00. The address is Töölönkatu 27.

Tourist information and activities in Helsinki

The city of Helsinki offers a lot to see and experience for visitors. The heart of Helsinki consists of Senate Square and Market Square. The National Museum of Finland, the Ateneum Art Museum as well as the Museum of Contemporary Art Kiasma are all within five minutes walking distance from there. Some of the other most popular sights in Helsinki include Suomenlinna Maritime fortress.

More information about activities in Helsinki can be found at

http://www.hel2.fi/tourism/en/matko.asp

2 Conference program

Tuesday, May 26th

08:45-09:00 Registration

09:00-10:00 Walter Schachermayer, University of Vienna:
The fundamental theorem of asset pricing for continuous processes under small transaction costs

10.00–10.30 Coffee break

10.30–12.00 Thilo Meyer-Brandis, University of Oslo:
Electricity spot price modeling with a view towards spike risk

Eckhard Platen, University of Technology, Sydney:
A benchmark approach to financial market modelling beyond semimartingales

Yuliya Mishura, Kyiv National Taras Shevchenko University:
Long-range dependence and non-semimartingale models in finance

12.00–13.30 Lunch

13.30–15.00 Tommi Sottinen, University of Vaasa:
Conditional small balls, local continuity and quadratic variation

Paolo Guasoni, Boston University :
The fundamental theorem of asset pricing for discontinuous processes

Rama Cont, Université Paris VI-VII:
Functional Itô formula and stochastic integral representation for functionals of semimartingales

15.00–15.30 Coffee break

15.30–17.00 Teemu Pennanen, Helsinki University of Technology:
Superhedging in illiquid markets

Alexander Kulikov, Moscow State University:
Pricing with multidimensional coherent risk measures

Ehsan Azmoodeh, Helsinki University of Technology:
European call option and fractional frictionless/friction Market

17.15 Get together

Wednesday, May 27th

09:00-10:00 Fransesco Russo, INRIA Rocquencourt and Université Paris 13:
Stochastic calculus via regularization and one application to mathematical finance

10.00–10.30 Coffee break

10.30–12.00 Andreas Basse, University of Aarhus:
When is a moving average a semimartingale?

Ivan Nourdin, Université Paris VI:
Itô's formulas in law for fractional Brownian motion

Jean Picard, Université Blaise Pascal:
A tree approach to calculus for non-semimartingales

12.00–13.30 Lunch

13.30–15.00 Christian Bender, Saarland University:
Approximating a geometric fractional Brownian motion and related processes via discrete Wick calculus

Giovanni Peccati, Université Paris Ouest:
Some limit theorems for functional of Gaussian or Poisson fields

Jürgen Schmiegel, Thiele Centre for Applied Mathematics in Natural Science:
Brownian semistationary processes and turbulence modelling

15.00–15.30 Coffee break

15.30–17.00 Alberto Ohashi, Ibmec São Paulo School of Business:
Weak approximations for Wiener functionals

Yeliz Yolcu Okur, University of Oslo:
SDE solutions in the space of smooth random variables

Mikko Pakkanen, University of Helsinki:
Stochastic Integrals and conditional full support

18.30 Sauna

Thursday, May 28th

09:00-10:00 Mark Podolskij, Federal Institute of Technology Zurich:
Power variation methods for stochastic processes

10.00–10.30 Coffee break

10.30–12.00 Bernt Øksendal, University of Oslo:
A general maximum principle for anticipating stochastic control and application to insider trading

José Corcuera, Universitat de Barcelona:
Multipower variation for Brownian semistationary processes

Jeanette Woerner, Technical University of Dortmund:
Fractional processes: Modelling and inference

12.00–13.30 Lunch

13.30–15.00 Monique Pontier, Institut Mathématique de Toulouse:
Optimal Strategies in a risky debt context

Hasanjan Sayit, Worcester Polytechnic Institute:
No arbitrage without semimartingales

Andreas Neuenkirch, Technische Universität Dortmund:
Milstein-type scheme without Lévy-area terms for SDEs driven by fractional Brownian motion

15.00–15.30 Coffee break

15.30–16.00 Salvador Ortiz, Universitat de Barcelona:
Itô-Stratonovich formula for Gaussian Processes: A Riemann sums approach

19.00 Conference dinner

3 List of participants

Organizers

Francesca Biagini, Universität München
José Manuel Corcuera, Universitat de Barcelona
Ari-Pekka Perkkiö, Helsinki University of Technology
Francesco Russo, Université Paris 13
Esko Valkeila (chair), Helsinki University of Technology

Invited speakers

Andreas Basse, University of Aarhus
Christian Bender, Saarland University
Rama Cont, Université Paris VI-VII
Paolo Guasoni, Boston University
Thilo Meyer-Brandis, University of Oslo
Yuliya Mishura, Kyiv National Taras Shevchenko University
Ivan Nourdin, Université Paris VI
Bernt Øksendal, University of Oslo
Giovanni Peccati, Université Paris Ouest
Eckhard Platen, University of Technology, Sydney
Mark Podolskij, Federal Institute of Technology Zurich (ETHZ)
Walter Schachermayer, University Vienna
Jürgen Schmiegel, University of Aarhus
Tommi Sottinen, University of Vaasa
Jeanette Woerner, Technische Universität Dortmund

Contributing speakers

Ehsan Azmoodeh, Helsinki University of Technology
Alexander Kulikov, Moscow State University
Andreas Neuenkirch, Technische Universität Dortmund
Alberto Ohashi, IBMEC São Paulo
Salvador Ortiz-Latorre, Universitat de Barcelona
Mikko Pakkanen, University of Helsinki
Teemu Pennanen, Helsinki University of Technology
Jean Picard, Université Blaise Pascal
Monique Pontier, Institut Mathématique de Toulouse
Hasanjan Sayit, Worcester Polytechnic Institute
Yelis Yolcu Okur, University of Oslo

Registered attendees

Nadia Belaribi, Université Paris 13
Mohamed Ben Alaya, Université Paris 13
Delphine David, Université d'Evry Val d'Essonne
Jasmina Djordjevic, University of Nis
Dario Gasbarra, University of Helsinki
Stefan Geiss, University of Jyväskylä
Mohamed Amine Ghezzar, Sonatrach
Cristina di Girolami, Université Paris 13
Stéphane Goutte, Université Paris 13

Harald Oberhauser, University of Cambridge
Ahmed Kebaier, Université Paris 13
Jukka Lempa, University of Oslo
Samuli Leppänen, Helsinki University of Technology
Lasse Leskelä, Helsinki University of Technology
Lavinia Ostafe, University of Vienna
Petteri Piiroinen, University of Helsinki
Anthony Reveillac, Humboldt Universität (Berlin)
Heikki Tikanmäki, Helsinki University of Technology
Olli Wallin, Ilmarinen Mutual Pension Insurance Company (Helsinki)
Lauri Viitasaari, Helsinki University of Technology

4 Abstracts

EUROPEAN CALL OPTION AND FRACTIONAL FRICTIONLESS/FRICTION MARKET

EHSAN AZMOODEH

Abstract

We first show that in the fractional Black-Scholes model all European options with convex payoff can be hedged perfectly and corresponding hedging strategy is as if the stock price had bounded variation. Moreover in the case of European call option, the hedging strategy *stop-loss-start-gain* is self-financing and hedging price is the same as one obtained in [2] and [3].
In the second part with proportional transaction costs to the model we study the asymptotic hedging problem in the case of European call option.

References

[1] Azmoodeh, E., Mishura, Y., and Valkeila, E. (2009). *On hedging European options in geometric fractional Brownian motion market model*. Submitted.
[2] Bender, C., Sottinen, T., and Valkeila, E. (2008). *Pricing by hedging beyond semimartingales*. Finance and Stochastics, **12**, 441-468.
[3] Valkeila, E. (2008). *On the approximation of geometric fractional Brownian motion*. HUT, Institute of Mathematics, Preprint A535, 2007..

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WHEN IS A MOVING AVERAGE A SEMIMARTINGALE?

ANDREAS BASSE

Abstract

Continuous time moving averages, as e.g. the fractional Brownian motion, the Ornstein-Uhlenbeck process and their generalizations, have been used repeatedly in finance, turbulence and related fields. The present talk is concerned with the question when is a moving average a semimartingale; various filtrations are considered. In particular, necessary and sufficient conditions are provided for a moving average to be a semimartingale when the driving process is a Brownian motion, a Lévy process or a Gaussian chaos process. To show these results we use and prove a new characterization of Gaussian chaos semimartingales.

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APPROXIMATING A GEOMETRIC FRACTIONAL BROWNIAN MOTION AND RELATED PROCESSES VIA DISCRETE WICK CALCULUS

C. BENDER

Abstract

The stochastic exponential $e^{B_t^H - t^{2H}/2}$ of a fractional Brownian motion B^H is intimately connected to the Wick product. On the one hand it admits a series representation in terms of Wick powers. On the other hand it solves a Doleans-Dade type stochastic differential equation with the integral interpreted in the fractional Wick-Itô-Skorokhod sense. We replace the fractional Brownian motion B^H by a weakly convergent approximation based on binary trials [3] and the Wick product by its rather simple discrete counterpart acting on the binary trials [2]. In this way we obtain approximations of $e^{B_t^H - t^{2H}/2}$ in terms of a discrete Wick power series and a discrete Wick difference equation. Weak convergence of both approximations is proved. Moreover, the approach is extended in order to establish weak Euler schemes for some systems of linear SDEs driven by a fractional Brownian motion in the fractional Wick-Itô-Skorokhod sense. In particular, we derive weak convergence of the discrete version of the fractional Black-Scholes market [1].

The talk is based on a joint work with Peter Parczewski (TU Braunschweig).

References

[1] Bender, C. and Elliott, R. J. (2004) Arbitrage in a discrete version of the Wick-fractional Black-Scholes market. *Math. Oper. Res.* **29**, 935-945.

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[3] Sottinen, Tommi (2001) Fractional Brownian motion, random walks and binary market models. *Finance Stoch.* **5**, (2001), 343-355.

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FUNCTIONAL ITO FORMULA AND STOCHASTIC INTEGRAL REPRESENTATION FOR FUNCTIONALS OF SEMIMARTINGALES

RAMA CONT & DAVID FOURNIE

Abstract

We present a *functional* extension of the Ito formula which allows to obtain explicit stochastic integral representations for non-anticipative functionals of continuous semimartingales with continuous dependence on the underlying path and its quadratic variation. Our formula extends recent work by B. Dupire to functionals which can depend on quadratic variation and allows for a natural interpretation in financial applications in terms of sensitivities. As a by-product, we show that a large class of functionals of a Brownian semimartingale can be described as solutions of a pathwise functional heat equation. This leads to model-free relations between sensitivities of path-dependent options.

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May 26-28, 2009, Helsinki University of Technology .

Multipower Variation for Brownian Semistationary Processes

José Manuel Corcuera

Abstract

In this work we study the asymptotic behaviour of power and multipower variations of processes Y:

$$Y_t = \int_{-\infty}^{t} g(t-s)\sigma_s W(ds) + Z_t,$$

where $g : (0,\infty) \to R$ is deterministic, $\sigma > 0$ is a random process, W is the stochastic Wiener measure, and Z is a stochastic process in the nature of a drift term. Processes of this type serve, in particular, to analyse data of velocity increments of a fluid in a turbulence regime with spot intermittency ?. The purpose of the present paper is to determine the probabilistic limit behaviour of the (multi)power variations of Y, as a basis for studying properties of the intermittency process σ. Notably the processes Y are in general not of the semimartingale kind and the established theory of multipower variation for semimartingales does not suffice for deriving the limit properties. Examples and an application to realised variance ratio are given.

This is a joint work with Ole E. Barndorff-Nielsen and Mark Podolskij.

THE FUNDAMENTAL THEOREM OF ASSET PRICING FOR DISCONTINUOUS PROCESSES

PAOLO GUASONI

Abstract

This talk discusses the Fundamental Theorem of Asset Pricing with transaction costs, when bid and ask prices are potentially discontinuous processes. The Robust No Free Lunch with Vanishing Risk (RNFLVR) condition for simple strategies is equivalent to the existence of a strictly consistent price system (SCPS). This result relies on a new notion of admissibility, which reflects future liquidation opportunities. The (RNFLVR) condition implies that admissible strategies are predictable processes of finite variation. We also develop an extension of the familiar Stieltjes integral for cadlag integrands and finite-variation integrators, which is central to modeling transaction costs with discontinuous prices.

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PRICING WITH MULTIDIMENSIONAL COHERENT RISK MEASURES

A. V. KULIKOV

Abstract

When we describe the portfolio consisting of some currencies it is more natural to use multidimensional approach given by Kabanov in [3]. The notion of multidimensional coherent risk measure was introduced in [2] by Jouini, Meddeb, Touzi. Their approach aims to take into account transactional costs while exchanging one currency to another. But in their model transactional costs are not random. So they do not take into account risk connected with changing of currency exchange rates that is one of the most important risks nowadays. In [4] we introduced the notion of multidimensional coherent risk measure which takes into account this type of risks and proved the representation theorem.

One of the most important problems of financial mathematics is pricing. The main result is the fundamental theorem of asset pricing (FTAP). There were various conditions in one-dimensional case to make the intervals of fair prices smaller. But these approaches are valid when we have basic asset or currency. However, it is not valid, for example, when we describe the portfolio consisting of some currencies.

The aim of this report is to give NGD condition for multidimensional random vectors. The technique given is analogical to one considered in [1], and the sets of fair prices are much smaller than with using NA pricing (See [3]). Here we give the notion and prove the theorems of No Good Deals (NGD) and define the set of fair prices using using this notion. Approach considered is analogical to [3], however the results obtained can be applied not only in discrete but also in continuous time. The sets of fair prices are smaller and we do not need to suppose the conditions like closeness of the set of strategies A. We give the notion of upper and lower prices and sub- and superhedging strategies along direction.

References

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[2] *E. Jouini, M. Meddeb, N. Touzi.* Vector-valued coherent risk measures. Finance and Stochastics, **8** (2004), p. 531–552.

[3] *Yu. M. Kabanov.* Hedging and liquidation under transaction costs in currency markets. Finance and Stochastics, **3** (1999), No. 2, p. 237-248.

[4] *A. V. Kulikov* Multidimensional coherent and convex risk measures. Probability Theory and Its Applications, **52** (2007), No. 4, p. 685–710.

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Electricity spot price modeling with a view towards spike risk

Thilo Meyer-Brandis

Abstract

The recent deregulation of electricity markets has led to the creation of energy exchanges, where the electricity is freely traded. We start by presenting the most salient statistical features of electricity spot prices with a particular attention to the European energy exchanges. These features can be adequately reproduced by the sum-OU model: a model representing the price as a sum of Levy-driven Ornstein-Uhlenbeck (OU) processes. We then present a new estimation method with particular emphasis on capturing the high peaks, which is one of the stylized features of such data. After introducing our method we show it at work for the EEX Phelix Base electricity price index.

LONG-RANGE DEPENDENCE AND NON-SEMIMARTINGALE MODELS IN FINANCE

YU. S. MISHURA

Abstract

Financial markets fairly often have a long memory and it is a natural idea to model them with the help of fractional Brownian motion or some of its modifications. However, it is not so straightforward to implement because the market model is appropriate when it does not admit arbitrage and the models involving fractional Brownian motion are not arbitrage-free.

The talk is devoted to some methods of construction of the long-memory arbitrage-free models and to the discussion of different approaches to this problem. In particular, we introduce the mixed Brownian–fractional-Brownian model and establish conditions that ensure the absence of arbitrage in such a model. Also we consider a fractional version of the Black–Scholes equation for the mixed Brownian-fractional Brownian model which contains pathwise integrals w.r.t. fBm, discuss possible applications of Wick products in fractional financial models and produce Black–Scholes equation for the fractional model involving Wick product w.r.t. fBm.

References

[1] Biagini, F., Hu, Y., Øksendal, B., Zhang T.: Stochastic Calculus for Fractional Brownian Motion and Applications. Probability and Its Applications, Springer (2008).

[2] Mishura, Yu.S.: Stochastic Calculus for Fractional Brownian Motion and Related Processes. Lecture Notes in Mathematics **1929**, Springer (2008).

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A MILSTEIN-TYPE SCHEME WITHOUT LÉVY-AREA TERMS FOR SDES DRIVEN BY FRACTIONAL BROWNIAN MOTION

A. NEUENKIRCH

Abstract

Recently, several Taylor-type approximation schemes have been proposed for stochastic differential equations (SDEs) driven by a fractional Brownian motion with Hurst parameter $H \in (1/4, 1)$, see e.g. *A.M. Davie, Differential equations driven by rough paths: an approach via discrete approximation, Appl. Math. Res. Express (2007), No. 2* and *P. Friz, N. Victoir, Multidimensional stochastic processes seen as rough paths, Cambridge University Press, to appear.* Since the distribution of the arising iterated integrals is known only in particular cases, the approximation of these integrals is required (as in the case of the classical Taylor schemes for SDEs driven by a standard Brownian motion).

In this talk, we will present a Milstein-type scheme that uses only increments of the driving fractional Brownian motion and show its convergence for $H > 1/3$. Moreover, we also discuss the exact rate of convergence and the asymptotic error distribution of this scheme.

This talk is based on a joint work with A. Deya and S. Tindel (both Université Henri Poincaré, Nancy).

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Itô's formulas in law for fractional Brownian motion

Ivan Nourdin

Abstract

It is well-known that fractional Brownian motion (fBm) of Hurst index H in $(0, 1)$ is not a semimartingale, except when $H = 1/2$ (standard Brownian motion case). In this talk, I will explain how recent results on the asymptotic behavior of weighted Hermite variations of fBm, proved by means of Malliavin calculus, allow to get Itô's formulas (in law) for fBm, when its Hurst index is either $1/4$ or $1/6$. It is based on several works, in collaboration with D. Nualart, A. Rveillac, J. Swanson and/or C. A. Tudor.

WEAK APPROXIMATIONS FOR WIENER FUNCTIONALS

ALBERTO OHASHI

Abstract

In this paper we introduce a simple space-filtration discretization scheme on Wiener space which allows us to study weak decompositions of some Wiener functionals and their differentiability with respect to the Brownian motion. In this setup, we obtain an *explicit robust* sequence of special semimartingales with respect to discrete-jumping filtrations which converges to a given weak Dirichlet process. The main novelty here is the approximation of erratic continuous processes (e.g. fractional exponentials) by means of a stochastic derivative operator on Wiener space introduced in this work.

As a by-product, we are able to approximate densities of square-integrable Brownian martingales in a very explicit way without assuming smoothness in the sense of Malliavin calculus. Applications of our abstract results to optimal stopping problems and martingale representation are also presented.

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Joint work with Dorival Leão, Universidade de São Paulo.

A GENERAL MAXIMUM PRINCIPLE FOR ANTICIPATING STOCHASTIC CONTROL AND APPLICATION TO INSIDER TRADING

BERNT ØKSENDAL

Abstract

In this talk we suggest a general stochastic maximum principle for anticipating stochastic differential equations driven by a Lévy type of noise. We use techniques of Malliavin calculus and forward integration. We apply our result to study a general optimal portfolio problem of an insider.

The talk is based on recent joint work with Giulia Di Nunno, Olivier Pamen and Frank Proske, all at CMA in Oslo.

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ITÔ-STRATONOVICH FORMULA FOR GAUSSIAN PROCESSES: A RIEMANN SUMS APPROACH

S. ORTIZ-LATORRE

Abstract

The aim of this work is to establish a change of variable formula for general Gaussian processes whose covariance function satisfies some technical conditions. The stochastic integral is defined in the Stratonovich sense using an approximation by middle point Riemann sums. The change of variable formula is proved by means of a Taylor expansion up to the sixth order and applying the techniques of Malliavin calculus to show the convergence to zero of the residual terms. The conditions on the covariance function are weak enough to include processes with infinite quadratic variation, and we show that they are satisfied by the bifractional Brownian motion with parameters (H, K) such that $1/6 < HK < 1$, and, in particular, by the fractional Brownian motion with Hurst parameter $H \in (1/6, 1)$.
This is a joint work with David Nualart.

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STOCHASTIC INTEGRALS AND CONDITIONAL FULL SUPPORT

MIKKO S. PAKKANEN

Abstract

We give a simple criterion for a stochastic process $Z := H + K \cdot W$, where H and K are respectively continuous and left-continuous processes independent of the driving Brownian motion W, which ensures that Z has the conditional full support property, introduced by Guasoni, Rásonyi, and Schachermayer [*Ann. Appl. Probab.* **18,** 491–520 (2008)] in connection to no-arbitrage criteria and superhedging in pricing models with transaction costs. As an application of this result, we show that several stochastic volatility models and mixed fractional Brownian motion have the conditional full support property.

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Research supported by the Academy of Finland and the Finnish Cultural Foundation.

SOME LIMIT THEOREMS FOR FUNCTIONALS OF GAUSSIAN OR POISSON FIELDS

GIOVANNI PECCATI

Abstract

We provide an overview of recent limit theorems involving non-linear functionals of Gaussian- or Poisson-based sochastic processes. These results have proven useful e.g. in the study of power variations associated with fractional processes, as well as of other limit objects that may be relevant in financial modeling.

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Superhedging in illiquid markets

Teemu Pennanen[*]

Abstract

We study superhedging of securities that give random payments possibly at multiple dates. Such securities are common in practice where, due to illiquidity, wealth cannot be transferred quite freely in time. We generalize some classical characterizations of superhedging to markets where trading costs may depend nonlinearly on traded amounts and portfolios may be subject to constraints. In addition to classical frictionless markets and markets with transaction costs or bid-ask spreads, our model covers markets with nonlinear illiquidity effects for large instantaneous trades. The characterizations are given in terms of stochastic term structures which generalize term structures of interest rates beyond fixed income markets as well as martingale densities beyond stochastic markets with a cash account. The characterizations are valid under a topological condition and a minimal consistency condition, both of which are implied by the no arbitrage condition in the case of classical perfectly liquid market models. We give alternative sufficient conditions that apply to market models with general convex cost functions and portfolio constraints.

[*]Helsinki University of Technology

A TREE APPROACH TO CALCULUS FOR NON-SEMIMARTINGALES

JEAN PICARD

Abstract

It is known that one can associate a real tree to any continuous real path ω. Our aim is to explain how some properties of ω can be read on its tree. In particular, it appears that the finiteness of the p-variations of ω is directly related to some geometric properties of the tree. This remark can be applied to the case where ω is a sample path of a stochastic process, and leads for instance to an estimator of the Hurst index of a fractional Brownian motion.

On the other hand, when ω has finite variation, the integrals $\int \rho \, d\omega$ can be written as Lebesgue integrals on the tree of ω, and it appears that these Lebesgue integrals have a sense even for some paths ω with infinite variation; our result contains the classical case of Young integrals, but also other cases. In particular, we can construct a calculus for the fractional Brownian motion coinciding with Lyons' rough path approach.

Reference

J. Picard, A tree approach to p-variation and to integration, *The Annals of Probability* **36** (2008), 6, 2235-2279.

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A BENCHMARK APPROACH TO FINANCIAL MARKET MODELLING BEYOND SEMIMARTINGALES

ECKHARD PLATEN

Abstract

This lecture introduces a general financial market modelling framework that allows to accommodate dynamics of asset prices that are not of semimartingale type, for instance, fractional Brownian motion driven models. The central building block of the framework is the best performing, strictly positive, tradable portfolio of the investment universe, the so called benchmark. This portfolio plays the role of the numeraire portfolio for derivative pricing and other risk management tasks. Important is the fact that the approach is assuming discrete trading and does not aim for reconciling continuous time limits for gains from trade as is possible in a semimartingale setting. A Diversification Theorem will be derived that allows to identify proxies of the benchmark in the real market. Empirical results show mono scaling properties for benchmarked securities but no multi-scaling is has been reported for various asset classes. A fractional minimal market model will be described that allows to model parsimoniously various stylized empirical facts of index securities.

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POWER VARIATION METHODS FOR STOCHASTIC PROCESSES

MARK PODOLSKIJ

ABSTRACT

In this talk we present an overview over power variation methods for various stochastic process that are used in finance or physics. Power variations is a powerful tool that provides solutions for many estimation and testing problems in the high frequency setting. In our talk we demonstrate asymptotic results for these objects and present various useful applications.

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OPTIMAL STRATEGIES IN A RISKY DEBT CONTEXT

MONIQUE PONTIER

Abstract

The paper analyses structural models for the evaluation of risky debt following H.E. LELAND [2], with an approach of optimal stopping problem (for instance cf. N. EL KAROUI [1]). Moreover we introduce an investment control parameter and we optimize with respect to the failure threshold and coupon rate. With investment, the failure threshold is lower and the rate coupon rises. Finally, we show that the value of the optimal coupon policy decreases and the level of bankruptcy rises if the strict priority rule is removed.

Keywords: corporate debt, optimal capital structure, default, optimal stopping.

References

[1] EL KAROUI N. (1981), *Les Aspects Probabilistes du Contrôle Stochastique*, Lecture Notes in Mathematics 876, p.73-238, Springer-Verlag, Berlin.

[2] LELAND H.E. (1994), "Corporate debt value, bond covenant, and optimal capital structure", The Journal of Finance, 49, 1213-1252.

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STOCHASTIC CALCULUS VIA REGULARIZATION AND ONE APPLICATION TO MATHEMATICAL FINANCE

FRANSESCO RUSSO

Abstract

Stochastic calculus via regularization (started by F.R and P. Vallois in 1991) is an approach of stochastic calculus beyond Itô calculus which has the following features: it constitutes a bridge between causal and non-causal calculus, it is suitable when the integrator is a non-semimartingale and it behaves similarly to a pathwise integral. It has applications in mathematical finance, fluiddynamics and irregular random media. In the first part of the talk (tutorial) we will recall the basic properties and the relations to Malliavin calculus. We will provide then a class of significant finite quadratic processes. Some comments about recent work (jointly with C. Di Girolami) on infinite dimensional extensions will be provided.

In the second part of tha talk some applications to basic mathematical finance will be discussed, corresponding to joint work with R. Coviello and C. Di Girolami. The stochastic integral intervening in the definition of self-financing property is forward integral (by regularisation). If one requires that a certain minimal class of investor strategies are self-financing, previous prices are forced to be finite quadratic variation processes. The non-arbitrage property is not excluded if the class $\mathcal{A}$ of admissible strategies is restricted. The classical notion of martingale is replaced with the notion of $\mathcal{A}$-martingale. A calculus related to $\mathcal{A}$-martingales with some examples is developed. Some applications to no-arbitrage, viability, hedging and the maximization of the utility of an insider are expanded. Hedging of path dependant options when the underlying is a finite quadratic variation process will be mentioned.

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No Arbitrage Without Semimartingales

Robert A. Jarrow[*] and Philip Protter [†] and Hasanjan Sayit [‡]

Abstract

We show that with suitable restrictions on allowable trading strategies, one has no arbitrage in settings where the traditional theory would admit arbitrage possibilities. In particular, price processes that are not semimartingales are possible in our setting, for example fractional Brownian motion.

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THE FUNDAMENTAL THEOREM OF ASSET PRICING FOR CONTINUOUS PROCESSES UNDER SMALL TRANSACTION COSTS

WALTER SCHACHERMAYER

Abstract

A version of the fundamental theorem of asset pricing is proved for continuous asset prices with small proportional transaction costs. Equivalence is established between: (a) the absence of arbitrage with general strategies for arbitrarily small transaction costs $\varepsilon > 0$, (b) the absence of free lunches with bounded risk for arbitrarily small transaction costs $\varepsilon > 0$, and (c) the existence of ε-consistent price systems – the analogue of martingale measures under transaction costs – for arbitrarily small $\varepsilon > 0$.

The proof proceeds through an explicit construction, as opposed to the usual separation arguments. The paper concludes comparing num?eraire-free and num?eraire-based notions of admissibility, and the corresponding martingale and local martingale properties for consistent price systems.

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BROWNIAN SEMISTATIONARY PROCESSES AND TURBULENCE MODELLING

J. SCHMIEGEL

Abstract

We discuss Brownian semistationary processes in the context of turbulence modelling. We focus on modelling the turbulent velocity field and the turbulent energy dissipation. In particular, we discuss semimartingale – nonsemimartingale issues and related inference problems.

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CONDITIONAL SMALL BALLS, LOCAL CONTINUITY AND QUADRATIC VARIATION

TOMMI SOTTINEN

Abstract

In the classical semimartingale-based option-pricing theory hedging and arbitrage are connected to *Equivalent Martingale Measures*. In the non-semimartingale option-pricing theory those measures do not exist. We propose that *Conditional Small Balls*, *Local Continuity*, and *Quadratic Variation* are the concepts one needs to understand hedging and arbitrage in this more general setup.

In this talk we discuss how hedges follow from Quadratic Variation, and how no-arbitrage follows from Conditional Small Balls and Local Continuity.

The talk is based on the work [1].

References

[1] Bender, C., Sottinen, T. and Valkeila, E. (2008) Pricing by hedging and no-arbitrage beyond semimartingales *Finance and Stochastics* **29**, 935-945.

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FRACTIONAL PROCESSES: MODELLING AND INFERENCE

JEANNETTE H.C. WOERNER

Abstract

Recently there have been proposed different methods for analyzing the empirical characteristics of high frequency financial data, e.g. increasing realized volatility, a certain correlation structure and paths regularity. Possible models capturing at least some of these features are additive market microstructure noise or components based on fractional Brownian motion. Another possibility would be to consider fractional Lévy processes instead which combine the desirable properties of Lévy processes with the correlation structure of fractional Brownian motion.

In the following we will first give some empirical results on the fine structure of high frequency data in terms of correlation and regularity of the sample paths. Then we will propose a new type of fractional Lévy process based on the kernel $|t-s|^{H-1/\alpha} - |s|^{H-1/\alpha}$, where α denotes the Blumenthal-Getoor index of the corresponding Lévy process. This choice of kernel unifies the approaches by Samorodnitsky and Taqqu (1994), Benassi at.al (2002, 2004) and Marquardt (2006).
(This work is partly based on joint work with Sebastian Engelke.)

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SDE SOLUTIONS IN THE SPACE OF SMOOTH RANDOM VARIABLES

Y. YOLCU OKUR

Abstract

Abstract. In this paper we analyze properties of a dual pair $(\mathcal{G}, \mathcal{G}^*)$ of spaces of smooth and generalized random variables on a Lévy white noise space. We show that the Fréchet algebra $\mathcal{G} \subset L^2(\mu)$ contains a larger class of solutions of Itô equations driven by pure jump Lévy processes. Further a characterization of $(\mathcal{G}, \mathcal{G}^*)$ in terms of the S-transform is given. We propose $(\mathcal{G}, \mathcal{G}^*)$ as an attractive alternative to the Meyer-Watanabe test function and distribution space $(\mathbb{D}_\infty, \mathbb{D}_{-\infty})$ [Watanabe, S.: On Stochastic Differential Equations and Malliavin Calculus. Tata Institute of Fundamental Research, Vol. 73, Springer-Verlag (1979)] to study strong solutions of SDE's. (Joint work with F. Proske and H. B. Salleh)

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